Supplementary Appendix to "Strategy-proofness in the Large" (for Online Publication)

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B Semi-Anonymity

Our main analysis considers anonymous mechanisms, where agents' outcomes depend on their own report and the distribution of all reports. The analysis generalizes straightforwardly, though at some notational burden, to the case of semi-anonymous mechanisms, as defined by Kalai (2004). In this setting, agents are divided into a number of groups, and agents within each group can be treated differently by the mechanism.

In this section, agents belong to **groups** g in a finite set G. The set of types is partitioned into subsets

$$T = T_{g_1} \cup T_{g_2} \cup \cdots \cup T_{g_G}.$$

A **semi-anonymous mechanism** is defined as $\{(\Phi^n)_{n \in \mathbb{N}}, (A_g)_{g \in G}\}$, where the A_g are the sets of actions available to each group g, and

$$A = A_{g_1} \cup \cdots \cup A_{g_G}$$

is the set of actions. As in the anonymous case, the Φ^n are functions

$$\Phi^n : A^n \to \Delta({X_0}^n).$$

The difference with respect to anonymous mechanisms is that agents in group g are restricted to play strategies in A_g. That is, if $t_i \in T_g$ then the support of any strategy $\sigma(t_i)$

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is contained in A_g. In a matching setting, for example, the groups may specify whether an agent is a man or a woman, and the agent's traits. Agents are then permitted to misreport their preferences over other match partners, but they cannot misrepresent their gender or their traits. **Limit mechanisms** are defined as in Section 3.1. In particular, we define limit mechanisms with respect to a single distribution $\mu \in \Delta T$, and not distributions of types within groups. Alternatively, one could assume that the number of agents in each group grows in a specific way, and that types are drawn i.i.d. within each group. We now formally define a two-sided matching mechanism, to clarify the definition.

Example B.1. (Two-Sided Matching) This example shows that semi-anonymous mechanisms include matching mechanisms in two-sided markets (Gale and Shapley, 1962). Agents are men and women, who differ on a set of traits. Groups g index both sex and the traits, so that the set of groups is

$$G = \{m_1, m_2, \cdots, m_M\} \cup \{w_1, w_2, \cdots, w_W\}.$$

That is, there are M groups of men and W groups of women. Men and women within each group have the same traits, and are equally good marriage partners. However, within each group, agents may differ in their preferences over the other groups. The way in which the semi-anonymous framework differs from the anonymous setting is that men and women may misreport their preferences, but cannot misreport their sex nor traits.

Formally, agent i's type is

$$t_i = (g_{t_i}, u_{t_i}),$$

where $g_{t_i} \in G$ is the agent's group, and u_{t_i} is a strictly positive utility function over the groups of the opposite sex. The set of outcomes $X_0 = G \cup \emptyset$. That is, each agent only cares about which type of man (woman) she (he) is matched to, or whether she (he) is unmatched. Utilities of each type t_i are given by $u_{t_i}(g)$ if she is matched to someone of the opposite sex. We extend u_{t_i} so that it is 0 if the agent is unmatched or matched to a group of the same sex.

Consider now a stable matching mechanism, using a tie-breaking lottery, as in school choice mechanisms (Abdulkadiroğlu et al., 2009). The mechanism is direct, so that $A_g = T_g$ for each $g \in G$. Men and women report a vector of types t, and therefore traits. This implies a weak preference ordering of each man over each woman and vice versa. The mechanism assigns a lottery number l_i to each agent, uniformly and independently distributed between 0 and 1. Lottery numbers are used to break ties between preferences. That is, preferences are refined to strict preferences, by using the lottery numbers to break ties. Conditional on a vector of lotteries l and a vector of reported types t, the mechanism implements a stable matching $x^n(t,l)$. The function $x^n(t,l)$ is taken to be symmetric, to conform to the semi-anonymity assumption. The mechanism is then defined as

$$\Phi^n(t) = \int_{l\in[0,1]^n} x^n(t,l)\,dl.$$

$\square$

Define a semi-anonymous mechanism as SP-L if no agent wants to misreport as a different type within the same group.

Definition B.1. *The direct, semi-anonymous, mechanism $\{(\Phi^n)_{\mathbb{N}}, T\}$ is* ***strategy-proof in the large (SP-L)*** *if, for any $m \in \bar{\Delta}T$ and $\epsilon > 0$ there exists n_0 such that, for all $n \geq n_0$, $g \in G$, and all $t_i, t_i' \in T_g$,*

$$u_{t_i}[\phi^n(t_i, m)] \geq u_{t_i}[\phi^n(t_i', m)] - \epsilon.$$

If the mechanism has a limit, this is equivalent to, for any $m \in \bar{\Delta}T$, $g \in G$, and all $t_i, t_i' \in T_g$,

$$u_{t_i}[\phi^\infty(t_i, m)] \geq u_{t_i}[\phi^\infty(t_i', m)].$$

Otherwise, the mechanism is ***manipulable in the large.***

The sufficient conditions for a mechanism to be SP-L also have straightforward extensions. The extension of the EF-TB condition is that no agent envies another agent in the same group, and with lower lottery number.

Definition B.2. *A direct semi-anonymous mechanism $\{(\Phi^n)_{\mathbb{N}}, (T_g)_{g\in G}\}$ is* ***envy-free but for tie-breaking*** *(EF-TB) if for each n there exists a function $x^n : (T \times [0,1])^N \to \Delta(X_0^n)$, symmetric over its coordinates, such that*

$$\Phi^n(t) = \int_{l\in[0,1]^n} x^n(t,l)dl$$

and, for all i, j, n, t, and l, if $l_i \geq l_j$, and if t_i and t_j belong to the same group, then

$$u_{t_i}[x_i^n(t,l)] \geq u_{t_i}[x_j^n(t,l)].$$

With this definition, an extension of Theorem 1 to semi-anonymous mechanisms follows from essentially the same proof.[1] This implies that the stable matching procedure in example B.1 is SP-L, because an agent envying another agent with a lower lottery number would violate the stability condition. In the working paper version of this article we extend a version of Theorem 2 to the semi-anonymous case.

C Details for Table 1

This Section provides supporting details for the classification of non-SP mechanisms presented as Table 1. For each mechanism we provide a formal definition of the mechanism in our setting, a formal proof of the classification, and relevant references.

C.1 Anonymous Mechanisms.

C.1.1 Multi-Unit Auctions

Example 1 in the main text proves that the uniform-price auction is SP-L and the pay-as-bid auction is not SP-L.

C.1.2 Single-Unit Assignment

In single-unit assignment problems, each agent is to be assigned at most one indivisible object, and there are no transfers. We refer the reader to Kojima and Manea (2010) and references therein for a detailed description of the environment and applications.

Formally, we define single-unit assignment as follows. Denote the set of object types by X_0. In a market of size n there are $\{q_{x_0} \cdot n\}$ units of object type x_0 available.[2] An agent of type $t_i \in T$ has a strict utility function u_{t_i} over X_0. It is assumed that X_0 includes a null object $\emptyset$, in supply $n - \sum_{x_0' \neq \emptyset} \{q_{x_0'} \cdot n\} \geq 0$, so that the total quantity of objects equals n. The utility of the null object is normalized to 0. Therefore, we assume that all agents strictly prefer any other object (termed a proper object) to the null object.

Boston Mechanism and Adaptive Boston Mechanism

[1] Lemma A.1 holds as is, since it is a statement about the empirical distribution of randomly drawn vectors of types, and therefore does not rely on the definition of a mechanism. Lemma A.2 holds for any two types t_i and t_i' in the same group, using the same proof, as for any such pairs of types the EF-TB condition in the semi-anonymous case implies the same properties as in the anonymous case. Given the two lemmas, the argument in the proof of Theorem 1 in Appendix A.1 holds as is, as long as we take t_i' to be in the same group as t_i, which is all that is needed for the definition of SP-L for semi-anonymous mechanisms.

[2] A bracketed expression denotes the nearest integer to the real number within brackets.

The Boston mechanism is a mechanism used in many cities to allocate seats in public schools. Abdulkadiroğlu and Sönmez (2003) show that the Boston mechanism is not SP, and Abdulkadiroğlu et al. (2006) document that it was extensively manipulated in practice. We now formally define the Boston mechanism and show that it is not SP-L. This complements an example given by Kojima and Pathak (2009), in a formally different environment, where the Boston mechanism can be manipulated in a large market. Here we consider the standard version of the Boston mechanism, as opposed to the simplified version used in our application in Section 6.

We now define the Boston mechanism. Fix a vector of reports t. To be consistent with the literature we will use the terminology of schools (the objects) and students (the agents). The mechanism first assigns to each student a lottery number l_i, uniformly and independently distributed in $[0, 1]$. The mechanism then proceeds in rounds, following the algorithm below.

1. The mechanism begins in $\texttt{round} = 1$. All students are initially unassigned.

2. Students that are still present in the mechanism take turns, in the order of their lottery number, with higher lottery numbers going first. In her turn, student i is permanently assigned to her $\texttt{round}^{\text{th}}$ choice, as given by u_{t_i}, if there are still seats in that school, or remains unassigned otherwise.

3. If all students have been assigned, finish, otherwise increase `round` by 1 and go to Step 2.

Note that the algorithm must finish, as eventually all students are assigned either to a proper school or to the null school $x_0 = \emptyset$. Therefore, conditional on a vector of types t and lottery numbers l the mechanism produces a well-defined outcome $x^n(t, l)$. Before lottery draws, the mechanism is defined as

$$\Phi^n(t) = \int_{l \in [0,1]^n} x^n(t, l) dl.$$

We now show that the Boston mechanism is not SP-L. Consider an economy with two proper schools, $x_0 = A, B$, and the null school $x_0 = \emptyset$, corresponding to being unmatched. That is, $X_0 = \{A, B, \emptyset\}$. Let $q_A = q_B = 1/6$. Consider a distribution $m \in \bar{\Delta} T$ such that $2/3$ of the agents prefer school A, while only $1/3$ prefer school B. Then, in a large market, the proper schools are filled in the first round with probability close to 1. Therefore, an agent has a negligible chance of getting her second choice. The chance of getting her first choice is $(1/6)/(2/3) = 1/4$ for school A and $(1/6)/(1/3) = 1/2$ for school B. That is, the limit mechanism is

$$\phi^{\infty}(t_i, m) = \begin{cases} \frac{1}{4} \cdot A + \frac{3}{4} \cdot \emptyset \text{ if } u_{t_i}[A] > u_{t_i}[B] \\ \frac{1}{2} \cdot B + \frac{1}{2} \cdot \emptyset \text{ otherwise.} \end{cases} \tag{C.1}$$

Note in particular that an agent who prefers school A faces a tradeoff when reporting her preferences. If she announces that she prefers school A, she will be assigned to it with 1/2 the chance she has of receiving school B. Therefore, it is not optimal for an agent with $u_{t_i}[A] > u_{t_i}[B] > u_{t_i}[A]/2$ to report truthfully.

Harless (2014) and Dur (2015) propose a variant of the Boston mechanism called the adaptive Boston mechanism. In the adaptive Boston mechanism, if a student points to a school where there are no seats left, then the student gets to point to the next school in her preference list (see Mennle and Seuken (2015) for a formal definition). The adaptive Boston mechanism is not SP-L. This is clear from our example above, because, in the example, schools A and B both run out of capacity in the first round.

Probabilistic Serial Mechanism

The probabilistic serial mechanism has been proposed as a solution to the assignment problem by Bogomolnaia and Moulin (2001). The mechanism works as follows. With time running continuously, agents "eat" probability shares of their favorite object, out of all objects still available. After probability shares of all objects are assigned, the objects are randomly assigned to agents according to these probabilities. We refer the reader to Kojima and Manea (2010) page 110 for a formal definition of the mechanism, as their analysis includes ours as a particular case.

Bogomolnaia and Moulin (2001) show that the mechanism is EF. Consequently, Theorem 1 guarantees that it is SP-L. Note that the fact that this mechanism is SP-L is a particular case of Kojima and Manea's Theorem 1.

Hylland and Zeckhauser Pseudo-Market Mechanism

Hylland and Zeckhauser (1979) proposed a pseudo-market mechanism for single-unit assignment, in which agents are endowed with equal budgets of an imaginary currency which they use to purchase probability shares of the objects. The mechanism works as follows. First, agents report their types, t. Second, the mechanism allocates each agent an equal budget $B > 0$ of an artificial currency. Third, the mechanism computes a competitive equilibrium price vector $p^* \in \mathbb{R}_+^{|X_0|}$ and a probabilistic allocation of goods to each agent. Each consumer's probabilistic allocation of goods is optimal given prices and the budget.

We refer the reader to the original paper for full details.

Hylland and Zeckhauser (1979) prove existence of competitive equilibria in a setting that is strictly more general than ours (in particular, they allow for indifferences). For each market size n and each possible reported vector of types $t \in T^n$, choose one such equilibrium in an anonymous manner, and use this equilibrium to define the resulting allocation $\Phi^n(t)$. As Hylland and Zeckhauser (1979) observe on page 307, since each agent has the same budget and faces the same prices, such a mechanism is EF. Consequently, Theorem 1 guarantees that it is SP-L.

C.1.3 Multi-Unit Assignment

In multi-unit assignment problems, each agent is to be assigned a finite number of indivisible objects. Transfers of a numeraire are not allowed. A prototypical application is the allocation of courses to students at business schools. For further details we refer the reader to Budish (2011).

Denote the finite set of object types by J. Each object j is available in supply $\{q_j \cdot n\}$. A bundle $x_0 \in X_0 = \mathcal{P}(J)$ specifies a subset of the object types.[3] A type t_i specifies a utility function u_{t_i} over bundles. We will adopt the terminology of course allocation, denoting object types by courses, and agents by students.

HBS Draft Mechanism

The mechanism used by Harvard Business School to allocate MBA courses was studied empirically by Budish and Cantillon (2012). Using survey data, they showed that students often misreport their preferences. Here we formally define the mechanism and show that it is not SP-L.

The HBS draft mechanism does not allow students to express preferences over bundles of courses. Instead, students submit a preference ordering over single courses. To examine the possibility of truthful reporting, we restrict our attention to preferences over bundles that are responsive to preferences over individual courses, with preferences over individual courses strict. We will say that a student of type t_i prefers course j_A to course j_B if she prefers a bundle consisting only of course j_A to a bundle consisting only of course j_B, that is, $u_{t_i}(\{j_A\}) > u_{t_i}(\{j_B\})$.

The HBS draft mechanism works as follows. First, each student is assigned a lottery number l_i, uniformly distributed in $[0, 1]$. In the first round, students take turns ordered by their lottery number, with higher lottery numbers going first. At her turn, student i chooses

[3] $\mathcal{P}(J)$ denotes the power set of J.

her favorite course out of the ones that are still available. In round two, the same procedure is repeated, but with students with lower lottery numbers going first. The procedure is repeated in the following rounds, with higher lottery numbers going first in the odd rounds and last in the even rounds. The mechanism ends after k rounds, where k is the number of courses required per student.

To see that this mechanism is not SP-L, consider the following example based closely on Example 1 of Budish and Cantillon (2012). There are 4 proper courses, $J = \{j_A, j_B, j_C, j_D\}$, of which students require $k = 2$ courses each. Each course has capacity for $\frac{2}{3}$ of the population, that is $q_j = \frac{2}{3}$ for each $j \in J$. Consider a probability distribution over students' reports where $\frac{1}{3}$ of the population lists courses in the order j_A, j_B, j_C, j_D, $\frac{1}{3}$ lists courses in the order j_B, j_A, j_C, j_D, and $\frac{1}{3}$ lists courses in the order j_A, j_C, j_D, j_B. Given this distribution of reports, the probability that course j_A reaches capacity either in the end of the first round, or early in the second round converges to 1, as the market grows large. Therefore, a student that ranks course j_A as her first choice has probability close to 1 of receiving it, while a student who ranks j_A second has probability close to 0 of receiving it. In contrast, course j_B is very likely to reach capacity either late in the second round, or early in the third round, in a large market. Consequently, a student who ranks course j_B either first or second is very likely to receive it. For this reason, a student whose true preference order is j_B, j_A, j_C, j_D profits by misreporting as j_A, j_B, j_C, j_D. By doing so, the student receives both j_A and j_B, her two favorite courses, rather than courses j_B and j_C if she reports truthfully.[4]

The Bidding Points Auction Mechanism

The bidding points auction mechanism is used by several business schools to allocate MBA courses. It has been described by Sönmez and Ünver (2010) and Krishna and Ünver (2008), who demonstrated that the mechanism is flawed in several important ways, despite its widespread use. We now define the bidding points auction mechanism and show that it is not SP-L.

The mechanism works as follows. Students report vectors of bids, with one bid per course. Students can only spend up to a budget of B points, so that the set of actions is the set of all vectors of bids that sum to at most B. We restrict the bids to be integers, so that

$$A = \{a_i \in \{0, 1, \cdots, B\}^J_+ : \sum_j a_{i,j} \leq B\}.$$

[4]This particular profitable misreport is valid for any cardinal preferences consistent with the ordinal preferences j_B, j_A, j_C, j_D. In other examples the profitability of a particular misreport might depend on cardinal preference information.

Given a vector of bids, the mechanism starts with the highest bid and allocates the course to the student, as long as the course still has capacity. Ties are broken randomly.

To examine the possibility of truthful reporting, we assume that students' preferences are additive, meaning that their utility for a bundle of courses is the sum of their utilities from the component courses in that bundle. This allows us to interpret a student's bid vector as an expression of their individual course preferences, and allows us to interpret the bidding points auction as a direct mechanism with $T = A$.

Consider the case where there are three courses, $J = \{j_A, j_B, j_C\}$. Consider an agent who likes the three courses j_A, j_B, j_C equally, and derives no utility of being unmatched. That is,

$$\begin{aligned} u_{t_i}(j_A) &= u_{t_i}(j_B) = u_{t_i}(j_C) = B/3, \\ u_{t_i}(\emptyset) &= 0. \end{aligned} \tag{C.2}$$

Consider a distribution of play m, such that, in the large-market limit, the last accepted bid for the courses j_A, j_B, j_C is $2B/3$ with very high probability. In that case, the agent should not report her true preferences, with bids equal to her utility. If bids are given by equation (C.2), then with very high probability the agent does not receive any course. If instead she bids B for one of the courses she likes, and 0 for the others, she receives at least one of the courses. Therefore, the mechanism is not SP-L.

Approximate Competitive Equilibrium from Equal Incomes (A-CEEI)

Budish (2011) proposed a pseudo-market mechanism for multi-unit assignment problems. Budish's setting is a strict generalization of ours. For that reason, we do not repeat all formal definitions, and refer the reader to the original paper for further details. In our setting, the A-CEEI mechanism can be defined as follows. First, assign each student a lottery number l_i uniformly and identically distributed in $[0, 1]$. Then give each student a budget in an imaginary currency of $1 + l_i \cdot \beta_{(n)}$, where $\beta_{(n)}$ is a strictly positive constant that is weakly decreasing in n, as defined in Budish (2011) page 1081. Budish's Theorem 1 guarantees that given these budgets there exists an approximate competitive equilibrium of the economy where agents purchase courses using the imaginary currency. The A-CEEI mechanism selects one such equilibrium, anonymously, and gives each agent his equilibrium allocation. This defines a function $x^n(\cdot, \cdot)$ giving an assignment of bundles $x^n(t, l) \in X_0^n$, for each vector of types t and lottery draws l. The A-CEEI mechanism is defined as

$$\Phi^n(t) = \int_{l \in [0,1]^n} x^n(t, l) dl.$$

To show that this mechanism is SP-L, we use Theorem 1. By the definition of approximate competitive equilibrium (Budish's Definition 1), after lotteries are drawn, no agent envies another agent with a lower lottery number. Therefore, the CEEI mechanism is EF-TB, and therefore SP-L.

The Generalized Hylland and Zeckhauser Pseudo-Market

Budish et al. (2013) have proposed an extension of the Hylland and Zeckhauser pseudo-market mechanism that can be used for multi-unit assignment problems. In the simplest setting they consider, students have additive preferences over courses. We therefore assume that T only includes additive preferences. With this assumption, their setting is a strict generalization of ours. Budish et al. (2013) then formally define the mechanism. It works similarly to the Hylland and Zeckhauser mechanism, with students purchasing probability shares of courses using a fake currency. The mechanism then calculates a competitive equilibrium allocation of probability shares. Finally, the mechanism implements a lottery over allocations that gives each agent her equilibrium probability share. Budish et al.'s Theorem 6 and Corollary 3 guarantee that the mechanism is well-defined, as both an equilibrium exists and can be implemented by a lottery over feasible assignments. Budish et al.'s Theorem 8 shows that the mechanism is envy-free. Along with our Theorem 1, this implies that the mechanism is SP-L.

C.1.4 Exchange Economies

Walrasian Mechanism

A Walrasian mechanism implements competitive equilibrium allocations in an exchange economy. Several contributions in the literature have considered approximate incentive compatibility of Walrasian mechanisms in large markets, including the classic paper by Roberts and Postlewaite (1976). We refer the reader to Jackson and Manelli (1997) for an overview and references. We note that this example has an infinite set of bundles X_0, which does not fit the framework in the body of the paper. However, the mechanism fits the more general framework considered in Appendix A.1.2, which allows us to use Theorem 1 to classify it as SP-L.

We consider an exchange economy with J goods. A type $t_i = (e_{t_i}, v_{t_i})$ specifies

- An endowment vector $e_{t_i} \in \mathbb{R}^J_+$.
- A continuous utility function v_{t_i} over bundles of goods in $\mathbb{R}^J_+$, taking values in $[0, 1]$.

Assume that the finite set of types T is such that, for any finite n and type vector $t \in T^n$, there always exists at least one competitive equilibrium where all agents of the same type receive the same bundle. This is guaranteed under standard assumptions on the set of utility functions and endowment vectors.

Given a type t_i, we define the utility function u_{t_i} over net trades $x_0 \in \mathbb{R}^J$ as

$$\begin{aligned} u_{t_i} &= v_{t_i}(e_{t_i} + x_0) \text{ if } e_{t_i} + x_0 \in \mathbb{R}^J_+ \\ & \quad -\infty \text{ if } e_{t_i} + x_0 \notin \mathbb{R}^J_+. \end{aligned}$$

We let X_0 be $\mathbb{R}^J$, the set of all possible vectors of net trades.

Having defined X_0 and T, we now define the mechanism. For all n, t, $\Phi^n(t)$ anonymously selects a competitive equilibrium allocation of an economy with the n agents of types in the vector t, such that agents of the same type receive the same bundle, and assigns each agent i her vector of net trades in that equilibrium.

Note that the Walrasian mechanism is EF, as each agent receives her preferred vector of net trades given prices. Furthermore, while X_0 is not finite, it does satisfy the more general assumptions in Remark 1. Namely, X_0 is a measurable subset of Euclidean space, utility is measurable and bounded above by 1, and the utility of telling reporting truthfully is at least 0. Therefore, by Theorem 1, the Walrasian mechanism is SP-L.

C.2 Semi-Anonymous Mechanisms

Semi-anonymity generalizes anonymity to allow a mechanism to treat agents differently if they belong to identifiably distinct groups. Examples include treating men and women differently in a matching mechanism, and treating buyers and sellers differently in a double auction. While the body of the paper deals with the notationally simpler case of anonymous mechanisms, semi-anonymous mechanisms are analyzed in Appendix B. This subsection classifies some of these mechanisms.

C.2.1 Double Auctions

Double auctions have been extensively studied as a simplified model of price formation. We consider auctions where buyers and sellers submit bids, and prices are given as the average of marginal winning and losing bids. See for example Rustichini et al. (1994) for further details and references.

Types t_i specify whether an agent is a potential buyer or seller, and a value. That is, types specify the agent's group, which is $g_{t_i} = b(\text{uyer})$ or $s(\text{eller})$, and her value for the object, which is v_{t_i}. Sellers are endowed with a unit of the object, while buyers are not. The set of types is $T = G \times V$, with $G = \{b, s\}$ and $V = \{1, \cdots, \bar{v}\}$. A bundle x_0 specifies whether the agent trades or not, with a dummy $d_{x_0} = 0$ or 1, and the price of the transaction

$$p_{x_0} \in P = \{(p' + p'')/2 : p', p'' \in V\}.$$

We have $X_0 = \{0, 1\} \times P$. Buyers and sellers have quasilinear utility. The utility of a bundle is 0 if the agent does not trade. If the bundle prescribes a trade, utility is $v_{t_i} - p_{x_0}$ for a buyer, and $p_{x_0} - v_{t_i}$for a seller.

The mechanism works as follows. Given t, let $n_s(t)$ be the number of sellers, and therefore the number of objects. The market clearing price is the average of the $n_s(t)^{\text{st}}$ and $n_s(t)+1^{\text{st}}$ highest valuations. The mechanism assigns bundles x_0 with this price to all agents. The objects are assigned to the agents with the $n_s(t)$ highest valuations, with uniform tie-breaking for agents tied with the lowest winning valuation. Formally, the mechanism $\Phi^n(t)$ assigns bundles x_0 specifying trade to all buyers with valuations higher than the price, all sellers with valuations lower than the price, and randomly rations agents with valuations equal to the price.

Note that the mechanism is envy-free. This is so because all agents pay the same price, and therefore do not envy the price paid by other agents. Moreover, at this price, agents who trade with probability 1 would rather trade than not trade, and likewise agents that trade with probability 0 would rather not trade. Agents that are rationed are indifferent between trading or not trading, and therefore the mechanism is envy-free.[5] Therefore, double auctions are SP-L.

C.2.2 Matching

This setting is defined formally in Section B, Example B.1. That section also defines stable matching mechanisms, which are shown to be SP-L using a semi-anonymous version of the EF-TB condition.

Priority Match

Priority match mechanisms are described by Roth (1991), who proved that these mechanisms can produce unstable outcomes. Roth also documented that labor market clearinghouses using priority matching mechanisms were very likely to fail, and hypothesized that the reason why they failed is that they produce unstable outcomes.

[5]Note that agents are only rationed in the case of a tie between the marginal winning and losing bids, and therefore both of these bids equal the price.

The priority match works as follows. Given a man i (woman) and a woman (man) j define the rank of i on j's preferences as 1 plus the number of men (women) who are strictly preferred to i. Assign to the pair i, j the priority $p_{i,j}$ equal to the rank of the man in the woman's preferences, times the rank of the woman in the man's preferences. The mechanism then proceeds by matching pairs with the lowest priorities first, breaking ties randomly.

To see that the priority match mechanism is not SP-L, consider the case where there is a single trait for men. Then women are indifferent over all men. In this case, the priority match mechanism coincides with the Boston mechanism, which is not SP-L.

It is interesting to note that Roth (1991) conjectured that the reason why stable matching mechanisms seem to succeed in practice, while priority matching mechanisms lead to unravelling and market failures, is stability. Our analysis, however, shows that stable matching mechanisms are SP-L, while priority matching mechanisms are not. Therefore, Roth's empirical finding can be phrased equivalently as saying that SP-L mechanisms succeed while non SP-L mechanisms fail.

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