INFORMATION, DISINFORMATION AND LOBBYING IN A MEDIAN VOTER MODEL

Gerald R. Faulhaber*

Abstract

Candidates run for (a one-shot) election, adopting a platform of a single-dimensional policy variable $p \in [0,1]$. Voters with heterogeneous single-peaked preferences over policies know candidates' announced platforms, but perceive that the actual policy which would be implemented should that candidate be elected is a random variable (with mean equal to the announced platform) whose distribution is common knowledge. Under certain restrictions, when candidates can announce any platform, a median voter equilibrium results. When candidates can costlessly modify the perceived distribution of actual policies around the announced platform policy, a median voter equilibrium also results, but under certain circumstances, candidates may choose an uninformative distribution in equilibrium. When modifying the perceived distribution is costly, we assume it is undertaken with the support of lobbying groups with an interest in the outcome. In general, we show that with two opposed lobbying groups, one will wish to provide more information to voters (reduce uncertainty) while the other will wish to provide disinformation to voters (increase uncertainty). If the strengths of the two groups are symmetric around the default distribution, then both will expend lobbying resources but no new net information will be provided. Under some conditions, the equilibrium results in greater uncertainty for both candidates than the default distribution.

* Professor, Public Policy and Management Department, Wharton School, University of Pennsylvania, Philadelphia, PA 19103. This research was conducted at the Institut d'Análisi Económica, Belleterra, Spain; the author is grateful for the Institut's support. The author thanks Carmen Matutes, Joan Estaban, Giacinta Cestone and Nicolas Melissas of the Institut for their comments and suggestions. E-mail: faulhaber@wharton.upenn.edu.

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1. Introduction

In previous work (1998), I considered a model in which voter/consumers voted for one of two candidates vying to be the regulator of a monopoly. The candidates' platforms were prices to be charged by the monopolist for each of two services. Voter/consumers (indexed by $q \in [0,1]$) had heterogeneous tastes for the two services and therefore different utilities over the electoral outcomes. Prices were determined in the median voter equilibrium of this electoral game. However, not all potential voters actually vote. Lobbyists who favored low prices for one service or the other were permitted to spend resources to increase the likelihood that voters who favored their position would actually vote.

The model was constructed to test two competing hypotheses about lobbying; Becker (1983) proposed that lobbying was a mechanism by which economic agents without political voice (such as firms) could press their case in the political arena. Thus, lobbying should increase the efficiency of political outcomes. Posner (1975) on the other hand notes that rent-seeking is a costly process, which costs may outweigh what efficiency benefits are realized. On the basis of simulation results, the paper concludes that the Posner effect appears to substantially outweigh the Becker effect.

However, the form of lobbying in the previous paper can be characterized as "get out the vote." Lobbying did not change people's minds or alter their preferences for candidates; it simply attempted to overcome the cost of voting by providing information on the importance of the issues. In this paper, I develop a model in which resources are expended to provide voters with information regarding candidate positions. Candidates are assumed to announce platforms, which is a policy selected from a single-dimensional continuous policy space. However, voters perceive that the actual policy that would be implemented should the candidate be elected is a random variable. The mean of this random variable is the announced platform, and its distribution is common knowledge. Voters know their own preferences as well as the platforms and distributions of all candidates, and make their voting decision based on expected utility.

I successively generalize the classic median voter model:

i. the distributions of the candidates are identical and cannot be changed; however, candidates can freely choose their announced platforms.

ii. Candidates can choose not only their platforms but also their distributions, by commitment devices, providing more information, etc. Changing their distribution from the default distribution is costless.

iii. Candidates can choose their platforms and their distributions, but it is costly to do so. Resources are provided by other agents, such as political parties, unions, firms, etc. that have an interest in the electoral outcome. I assume that there are two such agents with opposing interests in the policy dimension.

This last model constitutes the major contribution of the paper, in which I examine the outcomes of the lobbying game. I find that if interested parties expend resources to change the distributions of candidates, (i) disinformation, that is, increasing uncertainty, may well be the outcome; and (ii) the game is a prisoner's dilemma game, in which excessive resources are expended with relatively little effect on information outcomes. In fact, when the positions of the lobbying interest groups are identical, then in equilibrium the distributions of the candidates do not change even though resources are expended. The model is similar to certain advertising models, in which firms spend substantial resources on market-share increasing advertising which in equilibrium balance each other out.

SOMEPLACE AROUND HERE, PUT IN STUFF ABOUT RELATED WORK.

2. The Model

There are a continuum of voters $\theta \in [0,1]$ with distribution function $H(\theta)$, each of whom has preferences over policies $p \in [0,1]$, a single-dimensional policy space. As in previous work, this "policy" could be a price for a regulated monopolist's services, but in general it could be any single-dimensional policy space. Voter θ has a utility function $V_\theta(p)$ over policy space; we make no assumptions regarding the shape of this utility function, except we assume that it is single-peaked at $p_\theta = \arg\max_p V_\theta(p)$. Voter preferences $V_\theta(p)$ are common knowledge.

Much of the political economy literature assumes that voter preferences are concave over policies, sometimes making the stronger assumption of "Euclidean" preferences: $V_\theta(p) = [p_\theta - p]^2$. The assumption of concavity has appeal if the policies in question concern voters' incomes, such as income taxes and transfer payments, so that concavity has its usual connotation of risk aversion. Indeed, some have argued that *all* policy issues concern transfers. However, there are many policy issues in which income effects are negligible or zero. For example, incomes of voters are unlikely to be much affected by laws regarding abortion, environmental laws or laws regulating guns. Yet voters tend to have strong preferences on these issues, and there is no *a priori* reason to expect that these preferences are concave. Even in the case where regulators are deciding pricing policies for public utilities, prices are generally low enough that substitution effects predominate and income effects are negligible.[1] Therefore, we make no presumption (for the moment) about the shape of preferences, other than they are single-peaked. In fact, allowing preferences to be convex in certain intervals gives rise to some interesting results, as seen in subsequent sections.

There are two candidates, A and B, who are vying for a position to implement this policy. The candidates announce platforms p_A and p_B, which are the policies they assert they will implement if elected. However, voters perceive that the actual policy to be implemented should (say) candidate A be elected is a random variable with cdf F_{p_A} and pdf f_{p_A}, with a mean of p_A and support $[p_A-\Delta, p_A+\Delta] \subseteq [0,1]$. Both Δ and F are (for the moment) exogenous and common knowledge. The distributions are the same for both candidates but with different means: $F_{p_A}(p) = F_{p_B}(p+(p_B - p_A))$.

Voters vote their preferences, as defined by their expected utility, which is $\overline{V}_\theta(f_{p_A}) = \int_{p_X-\Delta}^{p_X+\Delta} V_\theta(p) f_{p_X}(p)dp$ for candidate X. There is no "strategic" or "sophisticated" voting in this one-shot game.

Endogenous Platforms, Exogenous Distributions.

In general, expected utility depends upon the entire distribution function; our maintained hypothesis, however, is that this function is exogenous and only the mean is endogenous. In this restricted game, expected utility is single-peaked under the following circumstances:

Condition C: $V_\theta(p)$ is concave on $[p(\theta)-2\Delta, p(\theta)+2\Delta]$. Throughout the remainder of the paper we assume that all $V_\theta(p)$ satisfy this condition.

Proposition 1: If C, then $\overline{V}(f_{p_X})$ is single-peaked in p_X for any f.

Proof: See Appendix.

Denote $\overline{p}_\theta(f_{p_X}) = \arg\max_{p_A} \overline{V}_\theta(f_{p_X})$. The Proposition ensures that this is uniquely defined on [0,1].

Since $\overline{V}(f_{p_X})$ is single-peaked, the game with endogenous p_X, fixed F and a single-dimension policy space has a unique equilibrium at the median voter's expected utility peak. We denote the median voter as $\hat{\theta}$, which solves $H(\hat{\theta}) = 0.50$.

Proposition 2: If C, the unique equilibrium to the electoral game with endogenous mean and fixed F is the median voter equilibrium, with $p_A = p_B = \overline{p}_{\hat{\theta}}(f_{p_X})$.

Proof: See Appendix.

Note that in general, the peak of expected utility $\overline{p}_q(f_{p_X})$ and the peak of utility p_q need not be equal.

Proposition 2 simply derives the classic median voter model, albeit under somewhat more stringent conditions than single-peakedness. It is interesting to note that multiple peaks can occur if concavity does not obtain in the full interval $[p_q-2D, p_q+2D]$. Thus, the size of the interval over which utility is concave appears to be a tight constraint.

Endogenous Distributions, Costless Adjustment.

Permitting candidates to choose the distribution F_{p_X} as well as the mean p_X enriches the electoral game structure. In the previous section, F_{p_X} was assumed exogenous, perhaps arising from voters' past experiences with politicians in a form of Bayesian updating. In this section, we assume that there is a default distribution F_{p_X} which voters assume if a candidate simply announces a platform. The idea is that when a candidate merely announces her platform, then voters assume the default distribution F_{p_X}. However, we also assume that each candidate has access to a technology for modifying this distribution, shaping voters' perceptions regarding the accuracy of her platform statements. In this section, we assume that these adjustments can be made without cost, just as statements announcing platforms can be made without cost. In the next section, we consider a model perhaps somewhat closer to reality, in which such adjustments are resource-consuming and must be financed, either by the candidate or her party, via agents with an interest in the electoral outcome. In this section, I permit candidates to costlessly choose both the mean and the shape of the distribution associated with their platforms.

The median voter model adopted here is the political equivalent of Bertrand competition in markets: competitive forces drive candidates to respond fully to voters' wishes. It might be expected, therefore, that in equilibrium candidates would be forced to reduce the uncertainty of their positions to maintain their share of risk-averse voters. In fact, this is in general not true; I show below that under certain circumstances, the median voter equilibrium actually involves *maximizing* the uncertainty of the platform distribution. However, under the usual (in this literature) assumption of concavity of utility, these expectations fulfilled. To show this, a proposition regarding extremal distributions is needed.

First, denote the degenerate distribution at p_X by

$$D_{p_X}(p) = \begin{bmatrix} 0 & p < p_X \\ 1 & p \geq p_X \end{bmatrix};$$

denote the Bernoulli distribution at $p_0 < ½ < p_1$ by

$$B_{p_0,p_1}(p) = \begin{bmatrix} 0 & p < p_0 \\ ½ & p_0 \leq p < p_1 \\ 1 & p_1 \leq p \end{bmatrix};$$

denote the set of all distributions on $[p_X\text{-D}, p_X\text{+D}]$ with mean p_X:
$\mathbf{F}_{p_X} = \{\text{distribution functions } F \mid E_F = p_X, F(p_X - \Delta) = 0, F(p_X + \Delta) = 1\}$.

Lemma: If $V_{\text{q}}(p)$ *is* $\left\{\begin{matrix}\text{concave}\\ \text{convex}\end{matrix}\right\}$ *in p, then* $\underset{F \in \mathbf{F}_{p_X}}{\arg\max} \int_{p_X-\Delta}^{p_X+\Delta} V_{\text{q}}(p) f(p) dp = \left\{\begin{matrix} D_{p_X} \\ B_{p_X-\Delta, p_X+\Delta}\end{matrix}\right\}$.

Proof: See Appendix.

That is, the degenerate distribution has the least uncertainty (viz., none) and the Bernoulli distribution has the most uncertainty. While the first part of this lemma is obvious, the second part is less so. Intuition might suggest that the uniform distribution on the interval $[p_X\text{-D}, p_X\text{+D}]$ is the least informative, but the result proves that it is indeed the extreme Bernoulli distribution.

Proposition 3: If $V_{\text{q}}(p)$ *is concave in p, then the unique equilibrium to the electoral game is* $p_A = p_B = p_{\hat{\text{q}}}$ *and* $F_{p_X} = D_{p(\hat{\text{q}})}$*; i.e., no uncertainty. Each candidate receives 50% of the votes.*

Proof: See Appendix.

Thus, voter uncertainty over candidates' "true" policies reduces to the median voter theorem with no uncertainty, under the rather strong conditions of costless distribution adjustment and concavity of voter preferences.

Unfortunately, neither of these assumptions is likely to be true in practice. As discussed above, there is no particular reason to assume that voter preferences are concave over the entire policy space unless the dominant feature of policies is income-related. We consider now how equilibrium outcomes are affected if concavity is not universal.

For any policy *p*, define the following sets:

$\text{Q}_{XL}(p)$ = {q | V_{q} is decreasing and convex on [p-2D, p-2D]}
$\text{Q}_{XR}(p)$ = {q | V_{q} is increasing and convex on [p-2D, p-2D]}
$\text{Q}_{NR}(p)$ = {q | V_{q} is increasing and not convex on [p-2D, p-2D]}
$\text{Q}_{NL}(p)$ = {q | V_{q} is decreasing and not convex on [p-2D, p-2D]}

Proposition 4: If $|\text{Q}_{XR}| > |\text{Q}_{NL}|$ or $|\text{Q}_{XL}| > |\text{Q}_{NR}|$, then the unique equilibrium of the electoral game is $p_A = p_B = p_{\hat{\text{q}}}$ and $D_{p_A} = D_{p_B} = D_{p_{\hat{\text{q}}}}$.

Proof: See Appendix.

The intuition of this proposition is that for one of the candidates (say the candidate to the left), the number of voters with convex preferences is larger than the number whose preferences are not convex. Adjusting your distribution to *D* increases your attractiveness to voters with convex preferences who would otherwise vote for your opponent ($= |\text{Q}_{XR}|$)while ensuring that the number of voters who would switch to your opponent (bounded above by the number voters that vote for you but have non-convex preferences (= | Q_{NL} |)) is less.

Why do convex preferences lead voters to prefer greater uncertainty? A simple example should illustrate. Suppose that contending politicians, in order to win the environmental vote, promise to pass a law restricting ordinary citizens from approaching any body of water closer than 20 meters (candidate A announces 19.999 meters), and they can commit to this position with certainty. Assume voter q has an ideal point a 0 meters; that is, she likes to sit directly by the water. Further, she receives no utility at any policy greater than 0 meters. If there is no uncertainty, this voter will receive no utility. If, however, candidate B now changes her distribution to $D_{0,40}$, then by voting for B, the voter stands to gain her ideal point of 0 meters with probability ½, which she clearly prefers.[2] Should candidate B adopt the distribution $D_{0,40}$, voter q would switch from candidate A to candidate B. If the number of such voters is larger than the number of voters who would switch from B to A as a result of this change, then the original distribution cannot be an equilibrium.

Under the hypothesis of Proposition 4, we again find a median voter equilibrium, but with a completely uninformative distribution of candidate policies. Again, the electoral model I employ forces candidates to respond to voter wishes; in this case, voters prefer more uncertainty to less, and that is what they get.

Do these results correspond to political behavior we observe in the real world? We certainly see how Proposition 3 works in practice: media typically press politicians to make commitments to positions on policy issues, presumably in response to voter demand. Voters may criticize a politician for not being sufficiently committed to their particular interests, perceiving some uncertainty they would prefer to avoid. Politicians often respond to these demands for more certainty; the George Bush promise of the 1988 US presidential election "Read my lips: no new taxes" was perceived as a strong personal commitment to tie the future president's hands (though, in the event, not an effective one).

But do we observe Proposition 4? In fact, politicians who have a reputation for taking a strong ideological position will often use a campaign to "soften" their image, indicating that they do indeed see more sides to an issue, and may be willing to listen to the arguments of others and change their position (once in office). In the context of this model, I would interpret such a candidate's initial position as having little platform uncertainty, perhaps because of voters' prior beliefs. However, this distribution may tie her hands and limit her ability to appeal to voters less committed to her positions. Therefore, the politician would attempt to increase uncertainty, in order to capture voters who would otherwise vote for her opponent. Some might view the initial position of George W. Bush as a "compassionate conservative" in the 2000 US presidential election as an extreme Bernoulli distribution, designed for minimum information content.

Costly Distribution Adjustment and Lobbying

In most economic activities reducing risk is not free, and there is no reason to expect that changing voters' perceptions of uncertainty in candidate platforms is any different. While announcing platform policies may be relatively costless, changing how voters think of a candidate is a more difficult and resource-consuming task. In general, I see that modifying voters' perceptions of candidate uncertainty can involve at least three types of costs:

(i) Direct costs in both time and money; examples include extra campaign time and media expenditures to convince voters of the candidate's position, encouraging endorsements from others, spelling out detailed plans to implement policies, commissioning books, articles and television specials to document either the staunchness or the flexibility of the candidate, etc.

(ii) Indirect current costs; examples include having other politicians endorse the candidate's position, or indicate their support for promised programs. This is likely to require the candidate adopt a position or make an endorsement of another politician that is costly.

(iii) Indirect future costs; examples include committing to another politician to support a program in the future (if elected) that is not in the candidate's principal interest, or agreeing to appoint a friend of a potential endorser to a key government position.

In this section, I focus on the direct costs, which generally involve financial resources that come from agents with an interest in the policy outcome, such as firms, unions, state or national political parties, trade associations, or single-issue lobbying groups such as environmental groups and gun groups. These groups may have an interest in the amount of uncertainty in candidate platforms (as we shall see below) and have the resources to fund the costs of adjusting candidate platform distributions.

I make several assumptions regarding the number, actions, and influence of these groups:

(i) There are two such groups, i = L,H, which are risk-neutral; group L prefers the lowest policy p possible, and group H the highest policy p possible. Their preferences can thus be represented as $V_X=a_X+b_Xp$, with $b_L<0<b_H$.

(ii) The candidates have no other sources of funds for changing voters' perceptions of their platform uncertainty; the groups therefore invest in changing perceptions of candidate uncertainty to suit their own interests. Candidates can respond to such changes by modifying their platforms.

(iii) The groups can invest in both candidates' uncertainty information. In general, we would expect groups to invest in both candidates, since their interest is not to elect a specific candidate but rather to have a particular platform adopted.

Why might an interest group invest to change voters' perceptions of candidate platform uncertainty? Recall that the equilibrium platforms announced by the candidates depend upon the distribution: $\bar{p}_{\hat{q}}(f_{\bar{p}})$. Different distributions *f* may lead to different p_X; since the interest groups are risk-neutral, they care only about the announced platform, and so make seek to influence it by changing *f*.

In order to understand the interest of groups in changing *f* it is necessary to understand how voter uncertainty affects the equilibrium platform. Proposition 5 shows how the relationship of the equilibrium platform to the median voter's ideal point depends on the asymmetry of the median voter's preferences around this ideal point. Proposition 6 demonstrates how the equilibrium announced platform is affected by changes in the uncertainty of the distribution.

First, note that the symmetry or asymmetry of a voter's utility function V_q depends V_q''', its third derivative. It is easy to show that if for all $p \in [p_q\text{-}D, p_q\text{+}D]$, $V_q''' \gtreqless 0$ then $V_q(p_q - p) \lesseqgtr -V_q(p_q + p)$. For V_q''' positive, there is more utility "mass" to the right of the peak $p(q)$ than to the left, and vice-versa for V_q''' negative. In the propositions below, we characterize asymmetry or symmetry by conditions on the sign of V_q'''.

Proposition 5: For any distribution f_{p_X} *and any* $q \in [0,1]$, $\bar{p}_q(f_{\bar{p}}) \lesseqgtr p_q$ as $V''' \lesseqgtr 0$ on $[p_X\text{-}D, p_X\text{+}D]$.

Proof: See Appendix.

Corollary: The equilibrium strategy p_X satisfies $p_X = \bar{p}_{\hat{q}}(f_{\bar{p}}) \lesseqgtr p_q$ as $V''' \lesseqgtr 0$ on $[p_X\text{-}D, p_X\text{+}D]$.

If the median voter's utility function is skewed left of her ideal point, then the equilibrium announced platform is to the left of the median voter's ideal point, and to the right of the ideal point if the skew is rightward.

Clearly, interest groups are affected by the degree of uncertainty in the distribution *f*. If the equilibrium outcome lies to the left of the median voter's ideal point, then group H would prefer to reduce uncertainty to zero, as this would move the equilibrium to the median voter's ideal point, which is closer to group H's ideal point. Proposition 5 and its corollary can be used to show this comparison between an uncertain and a certain outcome for one interest group. Is it possible to make comparisons between distributions with differing degrees of uncertainty?

In general, there is no general way to compare the uncertainty or risk in one distribution with that of another. Special assumptions, such as normality or logarithmic utility functions are generally required to impose a complete ordering on the set of all distribution functions. However, the partial ordering of second-order stochastic dominance is useful in dealing with general distributions, as we do here.

Recall the definition of second-order stochastic dominance: the distribution function *F* stochastically dominates the distribution *G*, denoted $F \succ_2 G$, iff

$\forall x, \int^{x} F(y)dy \le \int^{x} G(y)dy$ with strict inequality for at least one x. The principal result for our purposes is that for $E_F = E_G$ and $V''_{\theta} \lesseqgtr 0$, $\int V_{\theta}(p)dF(p) > \int V_{\theta}(p)dG(p) \Leftrightarrow F \underset{\prec_2}{\succ_2} G$.[3] If F stochastically dominates G, then all risk-averse agents prefer F to G; conversely, if all risk-averse agents prefer F to G, then F stochastically dominates G.

This relationship can be used as a measure of uncertainty: $F \succ_2 G \equiv F$ is *more certain* than G, or G is *more uncertain* than F. Of course, not all distributions can be compared in this way, so the ordering is not complete. However, the relationship can be used to determine how the equilibrium outcome responds to changes in the degree of uncertainty.

Proposition 6: For any family of distribution functions $F_{p_X}^{(z)}$ *on* $[p_X-\Delta, p_X+\Delta]$, *continuously differentiable in* $z\in[0,1]$, *such that* $z_1>z_2$ *iff* $f_{p_X}^{(z_1)} \succ_2 f_{p_X}^{(z_2)}$ *for all* p_X*; then* $\frac{dp_X}{dz} \gtreqless 0$ as $V'''_{\theta} \lesseqgtr 0$.

Proof: See Appendix.

Proposition 6 provides an essential tool we need to analyze the outcome of conflicting interest groups. For any family of distributions for which second-order stochastic dominance is a complete ordering, then decreasing uncertainty moves the equilibrium announced platform monotonically toward the median voter's ideal point, while increasing uncertainty moves the equilibrium announced platform monotonically away from the median voter's ideal point.

The final tool we need before setting up the lobbying game relates to candidate strategies when candidates can choose their platforms p_A, p_B, but have different distributions which they cannot change.

Proposition 7: Let F_{p_A} *be candidate A's distribution; if* $V_\theta''<0$ $\forall\, \theta \in$ *[0,1], then* $p_A = \bar{p}_{\hat{\theta}}(F_{\bar{p}})$ *is a dominant strategy for candidate A against any* F_{p_B} *such that* $F_{p_A} \succ_2 F_{p_B}$.

Proof: See Appendix

If the candidates have different distribution which can be ordered by second-order stochastic dominance, then the platform that maximizes expected utility of the median voter wins more than 50% of the vote against any platform with a more uncertain outcome. In general, there will be a continuum of dominant strategies for A; however, the median voter strategy is the only strategy which is dominant against *all* distributions stochastically dominated by F_{p_A}.

Proposition 7 provides the rationale for my assumption that a candidate with a distribution preferred to her opponent's will choose a platform which maximizes the expected utility of the median voter.

Having established the tools needed to analyze interest group behavior, I turn now to the interests of groups and why they are interested in these distributions of platform uncertainty. If an interest group is able to expend resources to move the equilibrium announced platform closer to its ideal point, then it is likely to spend resources to do so, *ceteris paribus.* For example, suppose the median voter's utility is skewed leftward, toward policies $p < p_{\hat{q}}$. Then $\overline{p}_{\hat{q}}(F_{\overline{p}}) < p_{\hat{q}}$; group H would like to reduce uncertainty, thus increasing the equilibrium platform. Group L would like to increase uncertainty, thus decreasing the equilibrium platform.

This conflict of interests between the two groups suggests a game formulation. I adopt a very simple formulation of the lobbying game, by which I mean expenditures of resources by interested (non-voting) agents to change the distribution of platform uncertainty of both candidates.

The Game

There is a set of voters $q \in [0,1]$ with distribution function $H(q)$ and preferences represented by utility functions $V_q(p)$ on a policy space $p \in [0,1]$. These utility functions satisfy condition C.

There are two candidates A and B who announce platform policies p_A and p_B, which they allege they will implement if elected. Voters perceive that there is *platform uncertainty*, that the "true" policy that each politician would implement if elected is a random variable with mean p_X and a distribution of platform uncertainty. Without further information, voters assume a default distribution of platform uncertainty, which is the same for both A and B. Voters vote their true preferences based on expected utility; candidates announce platforms that maximize their probability of winning. Everything is common knowledge: H, V_q, p_A, p_B, and the default distribution.

In accordance with Proposition 6, there is a family of probability distributions $F_{p_X}^{(z)}$ that are potential platform uncertainty distributions. This family $F_{p_X}^{(z)}$ is characterized by (i) mean equal to p_X, (ii) indexed by, and continuously differentiable in $z \in [0,1]$, (iii) with support $[p_X - D, p_X + D]$, (iv) $z_1 > z_2 \Rightarrow F_{p_X}^{(z_1)} \succ_2 F_{p_X}^{(z_2)}$, and (v) includes the extreme Bernoulli (most uncertainty), the single-point distribution (least uncertainty) and the default distribution:

$$F_{p_X}^{(0)} = B_{p_X - \Delta, p_X + \Delta}$$
$$F_{p_X}^{(½)} = \text{default distribution}$$
$$F_{p_X}^{(1)} = D_{p_X}$$

There are two interest groups, L and G, with linear utility functions $V_L = a_L + b_L p$ and $V_H = a_H + b_H p$, with $b_L < 0 < b_H$. These interest groups cannot vote, but may expend resources $x_L^A, x_L^B, x_H^A, x_H^B$ to modify the platform uncertainty distribution of each candidate to a new distribution in the family $F_{p_X}^{(z)}$. That is, lobbying expenditures may alter z for each candidate. The payoff function is $Z(x,x')$ with

$Z(0,0) = ½$	with no expenditures, the default distribution $z=½$ results;
$Z_1>0$, $Z_2<0$	lobbying expenditures x increase z, i.e., reduce uncertainty, while lobbying expenditures x' decrease z, i.e., increase uncertainty;
$Z_{11}<0$, $Z_{22}<0$	diminishing returns to lobbying;
$Z_{12}=Z_{21}<0$	expenditures decrease lobbying effectiveness of opponent.

This payoff function is the same for each candidate. Groups may choose to alter the distribution of either candidate or both.

The game is one-shot, in which interest groups choose expenditure levels, candidates choose platforms, and voters vote for candidates, all of whom have complete information, and rationally anticipate the strategies of the other players. I assume that $V_q'''<0$, so that voters' preferences are asymmetric around p_q, with $V_q(p_q-p)> V_q(p_q+p)$ (similar results obtain for $V_q'''>0$). For clarity of presentation, I also assume concave utility functions: $V_q''<0$. This pair of assumptions ensures that (i) for any distribution F, $\overline{p}_q(F_{\overline{p}}) \leq p_q$, so reducing risk increases expected utility, which is favored by group H; (ii) voters prefer risk reduction, i.e., for all voters, expected utility is increasing in z.

3. Results of the Model

From the assumption following Proposition 7, the winning platform is always the platform that maximizes the expected utility of the median voter. If both candidates have the same distribution, $z_A=z_B$, the equilibrium outcome is $\overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(z_A)}\right) = \overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(z_B)}\right)$. If candidate A has the preferred (by voters) distribution, $z_A>z_B$, the outcome is $\overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(z_A)}\right)$. In this case, candidate A wins outright and small increases in z_B have no effect on the result. In general, the outcome is $\overline{p}_q\left(F_{-}^{(\max\{z_A,z_B}\right)$.

a Nash equilibrium are:

$$\max_{x_H^a,x_H^B} a_H \quad b_H\,\overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(\max(z_A,z_B))}\right) \quad x_H^A - x_H^B\text{, with } z_A = Z\left(x_H^A, x_L^A\right), z_B = Z\left(x_H^B, x_L^B\right)\text{, and}$$

$$\max_{x_L^a,x_L^B} a_L + b_L\,\overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(\max(z_A,z_B))}\right) - x_L^A - x_L^B$$

The presence of a maximum (due to the "winner take all" nature of majority rule) within the maximands introduces a discontinuity into this maximization problem.

$$\frac{\partial \mathbf{L}}{\partial x_L^Y} = b_L \frac{d\overline{p}}{dz} \frac{d(\max(z_A, z_B))}{dz_Y} \frac{\partial Z(x_H^Y, x_L^Y)}{\partial x_L} \leq 1,\ x_L^Y \frac{\partial \mathbf{L}}{\partial x_L^Y} = 0,\ Y = A, B \text{, and}$$

$$\frac{\partial \mathbf{L}}{\partial x_H^Y} = b_H \frac{d\overline{p}}{dz} \frac{d(\max(z_A, z_B))}{dz_Y} \frac{\partial Z(x_H^Y, x_L^Y)}{\partial x_H} \leq 1,\ x_H^Y \frac{\partial \mathbf{L}}{\partial x_H^Y} = 0,\ Y = A, B$$

There are three possible types of equilibria:

i. $z_A > z_B = ½$ (or $z_B > z_A = ½$. In this case, both groups lobby to change the distribution of candidate A (or B) but not in her opponent. Group H is more effective than group L, either because it has higher marginal utility for z: $b_H > -b_L$, or because the technology of lobbying is favorable to increasing z: $Z_1 > Z_2$. It is unnecessary to change the distribution of B since A will win.

ii. $z_A = z_B < ½$. In this case, both groups lobby to change the distributions of both candidates. Group L is more effective than group H either because it has higher marginal utility for z: $b_H < -b_L$, or because the technology of lobbying is favorable to increasing z: $Z_1 < Z_2$. It is necessary to change both candidates distributions; if group L increases the uncertainty of candidate A beyond that of B, voters will choose B. Group L must therefore ensure that the uncertainty of both candidates is increased.

iii. $z_A = z_B \geq ½$. This somewhat anomalous equilibrium can occur if group H is more effective than group L but by very little. In this case, an equilibrium of type (i) is not supportable; group H would have an incentive to reduce uncertainty for candidate B, since $Z_1(0,0) > Z_1(x,x)$, which follows from $Z_{ii} < 0$ and $Z_{ij} < 0$.

We illustrate each of these equilibria with an example. For each example, assume that the lobbying technology is perfectly symmetric: $Z_1(x,x') = Z_2(x',x)$.

Type (i) equilibrium: $b_H >> b_L$. Only one candidate's distribution is adjusted (say, candidate A). There are two active first-order conditions; when equated, the familiar substitution relationship results: $\frac{b_L}{b_H} = \frac{\partial Z / \partial x_H^A}{\partial Z / \partial x_L^A}$. This equation coupled with $b_H >> b_L$ shows that $x_H^A >> x_L^A$. By symmetry of Z, we have $z_A >> z_B = ½$. Candidate A wins and adopts the strategy $p_A = \overline{p}_{\hat{q}}\left(F_{\overline{p}}^{(z_A)}\right)$. To show that neither group wishes to change the distribution for B, note first that group L has nothing to gain by increasing B's uncertainty, since at $z_A >> ½$, A wins. In order for group H to have an incentive to spend on reducing B's uncertainty, note that marginal expenditures have no effect on the electoral outcome. Group H must invest at least x_H', where $Z(x_H^A, x_L^A) = Z(x_H', 0)$ to have any effect at all.

Thus, in order for a defection to be profitable for H, we must have $b_H \frac{d\overline{p}}{dz} Z_1(x_H', 0) - x_H' >$

$b_H \frac{d\overline{p}}{dz} Z_1(x_H^A, x_L^A)$. Since by hypothesis, $x_H^A >> x_L^A$, it must be the case that x_H' is large and this inequality is not satisfied.

Type (ii) equilibrium: $b_H < b_L$. In this case, both candidates' distributions are adjusted. All four first-order conditions are active and the substitution equations indicate equal expenditures on both candidates, but higher expenditures for group L:

$$\frac{b_L}{b_H} = \frac{\partial Z / \partial x_H^A}{\partial Z / \partial x_L^A} = \frac{\partial Z / \partial x_H^B}{\partial Z / \partial x_L^B}$$, so $x_H^A = x_H^B > x_L^B = x_L^A$ and thus $z_A = z_B < ½$.

Type (iii) equilibrium: $b_H = b_L + e$, for small $e > 0$. This is the case of near-perfect symmetry between the two groups. A type (i) equilibrium cannot be supported; such an equilibrium involves nearly equal lobbying by both groups of candidate A's distribution ($x_H^A = x_L^A + e'$), which results in a very small change of $z_A = ½ + e''$. Group H would have incentive to lobby to reduce the uncertainty of B's distribution, since x_H' (which solves $Z(x_H', 0) = ½ + e'')$ is small and therefore $b_H \frac{d\overline{p}}{dz} Z_1(0,0) - x_H' > b_H \frac{d\overline{p}}{dz} Z_1(x_H^A, x_L^A) = 0$. A small increase in lobbying to reduce B's uncertainty is profitable for group H, even though it must pay the lump sum x_H' in order to realize the marginal benefit.

Of course, group L would find it just as profitable to respond by increasing uncertainty for candidate B as for candidate A, so it's response is the same for B as for A. Thus, the equilibrium involves both groups spending equally in both candidates, with H spending slightly more than L.

A special case on interest illustrates the nature of this game. Consider the base of perfect symmetry, with $b_H = b_L$. This results in a type (iii) equilibrium, in which both groups spend the same amount on both candidates. The net result is that the distributions do not change: $z_A = z_B = ½$. Nevertheless, there are lobbying expenditures; for $b_H >> 0 >> b_L$, these expenditures could be quite substantial, and yet the net result is no change. This illustrates the prisoners' dilemma nature of this game; both groups would prefer to spend zero and accomplish the same end while minimizing expenditures. However, (0,0) is not a Nash equilibrium of this game, as each has an incentive to cheat. The result is that possibly significant expenditures are made with the aim of changing the uncertainty of candidates' distributions, but the result is that no change occurs. In fact, the closer to parity are the two groups, the more likely is it that the outcome will involve quite small changes in the outcomes z_A, z_B relative to the amount of resources committed to this by the two groups.

A consequence of the model is that one lobbying group attempts to reduce uncertainty while the other group attempts to increase uncertainty. Which group does which depends upon the asymmetry of voters' preferences around their peak. The interest group that is disfavored by the asymmetry of preferences attempts to reduce uncertainty, while its competitor attempts to increase it. The lobbying equilibrium may well increase the platform uncertainty of both candidates, even if voters prefer less uncertainty (the converse is also true), if the group seeking more uncertainty is a more effective lobbyist (i.e., $b_H > -b_L$ or $Z_1 > -Z_2$).

Do such equilibria actually occur in practice? A very salient example occurred in the US when the Clinton Administration presented its health care plan in 1993 (although this example is about an issue, not a candidate, the principle is the same). When originally announced, the plan was complicated enough that few voters actually understood what was being proposed. The plan also changed during the public debate over it, adding further uncertainty to an already complex issue. Possible changes in health care coverage are likely to have substantial income effects, so it is plausible that voters' preferences are likely to be concave, leading to platform uncertainty aversion. In the event, the plan was opposed by a coalition headed by the insurance industry, which conducted a campaign that appeared designed to increase uncertainty regarding the proposal, perhaps to the point of providing disinformation. The Clinton Administration was not able to dispel this uncertainty by explaining and committing to voters exactly what the benefits and costs of the plan were. While both parties were strongly committed to achieving their ends ($b_H \approx b_L$), it appears it was easier to confuse voters than to clarify ($-Z_2 > Z_1$), and the opponents of the plan prevailed. The net result was very substantial expenditures by both parties (including the time and attention of the Administration, up to and including the President) with the net result being that nothing happened.

4. Conclusions

This paper presents a model in which voters perceive uncertainty about announced candidate platforms. We find:

i. If voter preferences are concave and candidates can freely choose both their announced platform and their distribution of platform uncertainty, then the unique pure strategy equilibrium is the classic median voter theorem, in which all uncertainty is removed.

ii. Under other conditions on voter preferences, a median voter equilibrium again results, but candidates choose the *most uncertain*, or *least informative*, distribution.

If adjusting the perceived distribution of candidate platform uncertainty is costly and competing interest groups control expenditures on such changes, then

iii. In this lobbying game, the equilibrium may involve both groups spending resources on both candidates; it may also involve spending on just one candidate.

iv. The lobbying game is a prisoners' dilemma game. If the interest groups are close to parity in their lobbying effectiveness, it is likely that the expenditures for lobbying are more significant that the outcomes would seem to warrant.

v. The equilibrium to the lobbying game may result in increasing the uncertainty of both candidates, even if voters prefer less uncertainty.

-- References--

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-- Appendix--

Lemma: $\dfrac{\partial f_{p_X}}{\partial p_X} = -\dfrac{df_{p_X}(p)}{dp}$

Proof of Lemma: Increasing p_A by Δp moves the entire distribution to the right by Δp, so that $f_{p_X+\Delta p}(p) = f_{p_X}(p-\Delta p)$; so $\dfrac{\partial f_{p_X}}{\partial p_X} = \lim\limits_{\Delta p \to o} \dfrac{f_{p_X}(p-\Delta p) - f_{p_X}(p)}{\Delta p} = -f'_{p_X}$.■

Proof of Proposition 1: We show that under the maintained hypothesis, $\dfrac{\partial \overline{V}_\theta(f_{p_X})}{\partial p_X} = -\int_0^1 V_\theta(p) f'_{p_X}(p)dp$ has a single root. Integrating by parts and noting that $f_{p_X}(0) = f_{p_X}(1) = 0$, we have $\dfrac{\partial \overline{V}_\theta(f_{p_X})}{\partial p_X} = \int_0^1 V'_\theta(p) f_{p_X}(p)dp$. It is immediately evident that $\dfrac{\partial \overline{V}_\theta(f_{p_X})}{\partial p_X}$ is $\begin{bmatrix} <, & p(\theta) < p_X - \Delta \\ >, & p(\theta) > p_X + \Delta \end{bmatrix}$. Again integrating by parts, we have for $p_\theta \in [p_X-\Delta, p_X+\Delta]$, $\dfrac{\partial^2 \overline{V}_\theta(f_{p_X})}{\partial p_X^2} = \int_0^1 V''_\theta(p) f_{p_X}(p)dp < 0$. Hence, $\dfrac{\partial \overline{V}_\theta(f_{p_X})}{\partial p_X}$ is monotonically decreasing on any interval containing p_θ. Therefore, $\dfrac{\partial \overline{V}'_\theta(f_{p_A})}{\partial p_A} = 0$ has a single root, denoted $\overline{p}_\theta(f_{\overline{p}})$.■

Proof of Proposition 2: Straightforward application of the Median Voter Theorem (Downs (1957).■

Proof of the Lemma: To simplify notation, we take the policy interval to be [0,1] with mean ½. Take a finite approximation to an arbitrary distribution, on the $(n+1)$ points $0=p_0<p_1<p_2<\ldots<p_n=1$, with probabilities $\pi_0,\pi_1,\pi_2,\ldots,\pi_n = \boldsymbol{\pi}$. I place no restrictions on the vector $\mathbf{p}$ other than $\Sigma\pi_i=1$ and $\Sigma\pi_i p_i=½$. We choose $\mathbf{p}$ to maximize expected utility:

$$\max_{\boldsymbol{\pi}} \mathcal{L} = \Sigma\pi_i V_\theta(p_i) - \lambda(\Sigma\pi_i - 1) - \mu(\Sigma\pi_i p_i - ½).$$

The first-order conditions are:

$$\frac{\partial \mathbf{L}}{\partial \mathsf{p}_i} = V_{\mathsf{q}}(p_i) - \mathsf{l} - \mathsf{m}p_i \le 0, \mathsf{p}_i \frac{\partial \mathbf{L}}{\partial \mathsf{p}_i} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \mathsf{l}} = \Sigma \mathsf{p}_i - 1 \ge 0, \mathsf{l} \frac{\partial \mathbf{L}}{\partial \mathsf{l}} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \mathsf{m}} = \Sigma \mathsf{p}_i p_i - ½ \ge 0, \mathsf{m}\frac{\partial \mathbf{L}}{\partial \mathsf{m}} = 0$$

Both constraints are active, so $\mathsf{l} > 0$ and $\mathsf{m} > 0$. Consider the (n+1) FOC on p_i; if any of them hold at equality, then for that i, the nonlinear function $V_{\mathsf{q}}(p_i)$ must be equal to the linear function $\mathsf{l} + \mathsf{m}p_i$. This can only occur on at most two points, say, $p_k \le p_j$.

If two points, $\mathsf{p}_k > 0$, $\mathsf{p}_j > 0$, and $\mathsf{p}_i = 0$, for $i \neq k,j$. Since the mean must equal ½, it must be that $p_k < ½ < p_j$. This pair of points determines the coefficients of the linear function $l(p; p_j, p_k)$: $\mathsf{m} = (V_{\mathsf{q}}(p_j) - V_{\mathsf{q}}(p_k)) / (p_k - p_j)$, and $\mathsf{l} = V_{\mathsf{q}}(p_j) - p_j(V_{\mathsf{q}}(p_j) - V_{\mathsf{q}}(p_k)) / (p_j - p_k)$. Expected utility is then $\mathsf{p}_j V_{\mathsf{q}}(p_j) + (1-\mathsf{p}_j) V_{\mathsf{q}}(p_k) = l(\mathsf{p}_j p_j + (1-\mathsf{p}_j) p_k; p_j, p_k) = l(0.50; p_j, p_k)$.

For a single point p_k, then $\mathsf{p}_k = 1$ and $\mathsf{p}_i = 0$, for $i \neq k$. Since the mean must equal ½, it must be that $p_k = ½$, in which case the distribution is $D_{½}$.

The first-order conditions restrict the extremal distributions to be either $D_{½}$ or B_{p_k, p_j}. First, assume V_{q} is concave. Then for all two-point solutions p_k, p_j, $E_D V_{\mathsf{q}} = V_{\mathsf{q}}(½) > p_k V_{\mathsf{q}}(p_k) + (1-p_k) V_{\mathsf{q}}(p_j) = E_B V$ so that $D_{½}$ maximizes expected utility over **F** for V_{q} concave.

Next, assume V_{q} is convex. Then $l(p; p_j, p_k) > V_{\mathsf{q}}(p)$ for all $p \in (p_k, p_j)$, so $E_D V_{\mathsf{q}} = V_{\mathsf{q}}(½) < l(½; p_j, p_k) = p_k V_{\mathsf{q}}(p_k) + (1-p_k) V_{\mathsf{q}}(p_j) = E_B V_{\mathsf{q}}$, so any two-point solution has higher expected utility than $D_{½}$, which therefore cannot be maximal. Consider the function

$$\ell(p) = \begin{cases} u(p) & 0 \le p \le p_k \\ l(p; p_k, p_j) & p_k \le p \le p_j \end{cases}$$

Then ℓ is convex on $[0, p_j]$; therefore, $l(p; 0, p_j) > \ell(p)$, for all $p \in (0, p_j)$, so $l(0.5; 0, p_j) > \ell(0.5) = l(0.5; p_j, p_k)$ = expected utility at points p_j, p_k. Next, consider the function

$$\bar{\ell}(p) = \begin{cases} l(p; 0, p_j) & 0 \le p \le p_j \\ V_{\mathsf{q}}(p) & p_j \le p \le 1 \end{cases}$$

This is a convex function on [0,1]; therefore $l(p; 0,1) < \bar{\ell}(p)$, for all $x \in (0,1)$, so that the expected utility at points 0,1 is $l(0.5; 0,1) > \bar{\ell}(0.5) > l(0.5; p_j, p_k)$ = expected utility at points x_j, x_k. Hence, expected utility is maximized at $x_k = 0$, $x_j = 1$, so that $B_{0,1}$ maximizes expected utility over **F** for V_{q} convex.■

Proof of Proposition 3: Assume not; then $p_A \neq p_B \neq p_{\hat{q}}$ and $D_{p_A} \neq F_{p_a} \neq F_{p_B} \neq D_{p_B}$. Wlog, assume $p_B > p_{\hat{q}}$, and let $\tilde{q}$ solve $p_B = \bar{p}_q(F_{\bar{p}})$. Then candidate A can adopt a strategy of $p_A = p_B - e$ and $F_{p_a} = F_{p_B}$, and win $H(\tilde{q}) > H(\hat{q}) = 0.50$ of the vote, so this cannot be an equilibrium. Assume that $p_A = p_B = p_{\hat{q}}$ and $D_{p_A} \neq F_{p_a} \neq F_{p_B} \neq D_{p_B}$. This cannot be an equilibrium since candidate A can choose D_{p_A}, a distribution which is preferred by all voters to F_{p_B}, so that candidate A gets 100% of the votes. From Proposition 2, if both candidates chose D_{p_q}, then both candidates achieve 50% of the votes. Finally, if $p_A \neq p_B \neq p_{\hat{q}}$ and $D_{p_A} = F_{p_a}, F_{p_B} = D_{p_B}$, then by Proposition 2 this cannot be an equilibrium. Since D_{p_q} is maximal in the set F of all distributions, this is the unique equilibrium.

Proof of Proposition 4: As in the previous proof, it must be that $F_{p_A} = F_{p_B}$, with $p_A = p_B = \bar{p}_{\hat{q}}(F_{\bar{p}})$. If not, then the argument in the above proof demonstrates that this is not an equilibrium. Now assume $F_{p_A} = F_{p_B} \neq B_{\bar{p}_{\hat{q}}(B)-\Delta, \bar{p}_{\hat{q}}(B)+\Delta}$. If either candidate (say, the candidate of the left) adjusts their distribution to $B_{\bar{p}_{\hat{q}}(B)-\Delta, \bar{p}_{\hat{q}}(B)+\Delta}$, then she gains $|Q_{XR}|$ votes while losing less than $|Q_{NL}|$ votes. Since $|Q_{XR}| > |Q_{NL}|$, this is a winning strategy. Therefore, this can't be an equilibrium. Note that since $\bar{p}_q(F) \in [p_q - \Delta, p_q - \Delta]$ for all $F \in \mathbf{F}$ and all $q \in [0,1]$, then from the definition of Q_{XR} all voters $q \in Q_{XR}$ have convex preferences on $[\bar{p}_{\hat{q}}(F) - \Delta, \bar{p}_{\hat{q}}(F) + \Delta]$ for all $F \in \mathbf{F}$. Since $B_{\bar{p}_{\hat{q}}(B)-\Delta, \bar{p}_{\hat{q}}(B)+\Delta}$ is maximal in the set $\mathbf{F}$ of all distributions for V_q convex, $F_{p_A} = F_{p_B} = B_{\bar{p}_{\hat{q}}(B)-\Delta, \bar{p}_{\hat{q}}(B)+\Delta}$ is the unique equilibrium. ■

Proof of Proposition 5: Evaluating the derivative of expected utility at $p(q)$ yields
$$\left.\frac{\partial \bar{V}}{\partial p_A}\right|_{p_X = p(q)} = \int_{p_A-\Delta}^{p_A+\Delta} V_q'(p) f_A(p)dp \gtreqless V_q'\left(\int_{p_A-\Delta}^{p_A+\Delta} p f_A(p)dp\right) = V_q'(p_q) = 0 \text{ as } V''' \gtreqless 0,$$
from Jensen's inequality. Since $\bar{V}$ is single-peaked, then $\bar{p}_q(f_{\bar{p}}) \gtreqless p_q$ as $\left.\frac{\partial \bar{V}}{\partial p_A}\right|_{p_X = \bar{p}} \gtreqless 0$. ■

Proof of Proposition 6: Totally differentiate the first-order condition for $\bar{p}_q(F_{\bar{p}})$:
$$\frac{\partial \bar{V}}{\partial p_X} = \int_{p_X-\Delta}^{p_X+\Delta} V_q'(p) f_{p_X}^{(z)}(p)dp = 0.$$
We obtain
$$[V_q'(p_X+\Delta) f_{p_X}^{(z)}(p_X+\Delta) - V_q'(p_X-\Delta) f_{p_X}^{(z)}(p_X-\Delta)]dp_X$$
$$+\left[\int_{p_X-\Delta}^{p_X+\Delta} V_q'(p)\frac{\partial f_{p_X}^{(z)}}{\partial z}dp\right]dz = 0.$$
Thus,

$$\frac{dp_X}{dz} = -\frac{\int_{p_X-\Delta}^{p_X+\Delta} V_\theta'(p)\frac{\partial f_{p_X}^{(z)}}{\partial z}dp}{V_\theta'(p_X+\Delta) f_{p_X}^{(z)}(p_X+\Delta) - V_\theta'(p_X-\Delta) f_{p_X}^{(z)}(p_X-\Delta)}.$$

Since $p(\theta) \in [p_X-\Delta, p_X+\Delta]$, $V_\theta'(p_X+\Delta) < 0 < V_\theta'(p_X-\Delta)$ and $f_{p_X}^{(z)} \geq 0$, so the denominator is always negative. Since $f_{p_A}^{(z_1)} \succ_2 f_{p_A}^{(z_2)}$ for all $z_1 > z_2$, we have

$$\left[\int_{p_X-\Delta}^{p_X+\Delta} V'(p;\theta)\frac{\partial f_{p_X}^{(z)}}{\partial z}dp\right]dz = \left[\frac{1}{\Delta z}\int_{p_X-\Delta}^{p_X+\Delta} V'(p;\theta)[f_{p_X}^{(z+\Delta z)}(p) - f_{p_X}^{(z)}(p)]dp\right]dz \gtrless 0 \text{ as } V_\theta''' \lesseqgtr 0.$$

Hence, $\frac{dp_X}{dz} \gtrless 0$ as $V_\theta''' \lesseqgtr 0$. ■

Proof of Proposition 7: To be supplied.

-- Notes --

[1] An argument can be made that preferences that are convex over large intervals of policy space could well be the norm. Suppose for example that voters are highly ideological in that they each believe there is a "right" policy, and that all other policies, even ones quite close to their ideal point $p(q)$, are very bad. These preferences would be sharply peaked at $p(q)$ and convex in the interval $(p(q),1]$ as well as the interval $[0,p(q))$. Groups of voters that claim special knowledge about policy may have similar preferences. For example, many economists would argue strongly that the correct trade policy for a country is free trade, or that the correct environmental policy is tradable pollution permits, and take a dim view of close substitutes for either of these policies.

[2] This specific utility function violates Condition C; it is straightforward but notationally confusing to construct an example of this type that does satisfy Condition C.

[3] This result is usually stated as, for $V_q' > 0$ and $V_q'' < 0$, $\int V_q(p)dF(p) > \int V_q(p)dG(p) \Leftrightarrow F \succ_2 G$. The condition that $V_q' > 0$ can be dispensed with if the means are equal. In this paper, I am concerned with both convex and concave utility functions, so I include both in the proposition.