

# ROLLING ELEMENT BEARING DESIGN THROUGH GENETIC ALGORITHMS

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## *Abstract-*

The design of rolling element bearings has been a challenging task in the field of Mechanical Engineering. Traditional approaches to the design optimization of such bearings have proved to be computationally time intensive and have yielded solutions that are yet to be theoretically proven optimal. While most of the real aspects of the design are never disclosed by bearing manufacturers, the common engineer is left with no other alternative than to refer to standard tables and charts containing the bearing performance characteristics. This paper presents a more viable method to solve this problem using *Genetic Algorithms* (GAs). Since the algorithm is basically a guided random search, it weakens the chances of getting trapped in local maxima or minima. The paper also highlights the superiority and the ease of using GAs for the design as also the success of churning solutions that are far better than those obtained by conventional techniques.

*Keywords-* Rolling element bearings, Optimum design, Genetic algorithms.

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## 1. INTRODUCTION

In general a bearing is a machine element that plays a vital role in transferring loads between two moving machine parts. The term *Rolling element bearing* is used to describe that class of bearings wherein the main load is transferred through elements in rolling contact rather than in sliding contact. Rolling element bearings are very popular because of their low starting torque and kinetic friction. Rolling element bearings have wide applications right from home appliances to industrial machines such as gear boxes, electric motors, instruments and meters, internal combustion engines, agricultural industries, textile industries, aircraft gas turbines, etc.

The design of rolling element bearings has a great impact on the performance, life and reliability of bearings. Consequently, it also affects the operating quality and economisation of machines on which the bearings are used. The responsibility of selecting an optimum scheme from all possible alternative designs, or more simply stated to make the best choice rests solely on the bearing designer.

The widespread use of rolling element bearings [2] has prompted many a researcher to look into simpler and naïve methods of computing optimal values of their associated design parameters. Conventional methods [13] are deterministic in nature and use only a few geometric design variables because of their complexity and convergence problems.

Refinements in the design can be made only by taking into account adequate number of geometric design variables and their associated constraints defined by the boundary dimensions, the mobility and the strength of the bearing parts. It is this increase in design variables that tends to make the problem complex and computationally time consuming. Worse, there is no way of concurring whether the solutions derived are optimal ones or those that were led astray into local maxima. In other cases the problem may not converge to a solution.

The best design scheme has optimum technical and economic indices. For this purpose, various methods are adopted. A method of *intuitional optimization* means selecting the best from several designing schemes while a method of *testing optimization* means modifying a design by sample tests gradually in order to seek the best one. Also a method of *evolving optimization* represents the renewal of a design for improvement upon comments of customers or users. Mathematical optimization methods have an edge over all the above techniques in that they are less time consuming and have better economic characteristics. Developments in computer technology have proved to be a great boon to the world of mathematical optimization. Computer-aided design (CAD), a dynamic and a relatively new field of engineering science, is in fact based on the mathematical optimization.

## **1.1 Motivation behind using Genetic Algorithms**

The conventional deterministic optimization techniques have inherent slow convergence. They also suffer from local minima (or maxima) problems and their complexity increases drastically with increase in number of design parameters. Among the stochastic optimization techniques, Genetic Algorithms (GAs) [1,4,5,8,11,14,16] are the most popular and promising ones that overcome all the aforementioned hurdles. One of the advantages of GAs is that they explore different areas of the search space in parallel and have no presumptions with regard to the search space. Better still, they provide several alternative solutions to the problem at hand and can be easily combined with other methods.

## **1.2 Genetic Algorithms in Mechanical Engineering**

The use of GAs in the realm of Mechanical Engineering has been limited. The algorithms remain unexploited except in a few cases [6,7,10,12]. In the field of Computer Aided Design (CAD), an area very much akin to Mechanical Engineering, GAs use feedback from the evaluation process to select the fitter designs, generating new designs through the recombination of parts of the selected ones. Eventually, a population of high performance designs is created. Choi and Yoon [3] have used GAs to design an automotive wheel-bearing unit with discrete design variables. The authors are not aware, to date, of any literature available on the use of GAs for designing rolling element bearings with continuous design variables.

Standard nonlinear programming techniques to solve the discrete optimization problem [13] are inefficient and computationally expensive. Also, in most cases, such techniques find a local optimum that is closest to the starting point. In contrast, GAs are well suited to solve such problems and in most cases can find the global optimum with a high probability since a population of design points, rather than a single point, is used for starting the procedure.

### **1.3 Basic Overview of Genetic Algorithm**

The algorithm is provided with a set of possible solutions (represented by *chromosomes*) termed as a *population*. Solutions from one population are taken and used to form a new population. This is motivated by a hope that the new population will perform better than its predecessors. Solutions chosen to form new solutions (*offsprings*) are selected based on their *fitness* - the more suitable they are the better their chances of being reproduced. This process of selection is repeated till some predetermined condition based on, for instance, the number of populations or improvement of the best solution, is satisfied.

This paper describes an attempt to solve the design optimization problem for rolling element bearings with five design parameters using GAs based on the requirement of the longest fatigue life. This is a singularly difficult problem and hardly any satisfactory solution has been reported in the literature before this effort [2]. The problem, if solved by conventional methods, would possibly never converge.

## 2. DESIGN OPTIMIZATION PROBLEM OF ROLLING ELEMENT BEARINGS

On the basis of operating requirements, different objective functions for rolling element bearings may be proposed, the most important of these being the requirement of the longest *fatigue life*. In normal operating conditions of rolling element bearings, the main mode of failure is *contact fatigue*. Without a special note, the bearing life means contact fatigue life. The basic requirement for conventional rolling element bearings is thus a long fatigue life. To solve this problem for a given size of the bearing outline or *boundary dimensions* (i.e. bearing bore,  $d$ , and outside diameter,  $D$ ), the *dynamic load rating*  $C$  should be maximum. The dynamic load rating  $C$  is defined as the constant radial load which a group of apparently identical bearings can endure for a rating life of one million revolutions of the inner ring (stationary load and stationary outer ring). The fatigue life [15],  $L$ , of the bearing (in millions of revolutions) subjected to any other applied load  $F$  is given by

$$L = (C / F)^a \quad (1)$$

where  $a = 3$  in the present case (for ball bearings).

### 2.1 Design Parameters

Design parameters are basically internal structure dimensions (see Figures 1 & 2) and other variables, called main parameters, which need to be determined in the bearing design. The five design parameters [2] for the given problem are represented as

$$X = [D_b, Z, D_m, f_o, f_i]^T \quad (2)$$

where

$D_b$  = Diameter of the ball

$Z$  = Number of balls

$D_m$  = Pitch diameter (i.e. diameter of locus of center of the balls)

$f_o = r_o/D_b$  = Curvature radius coefficient of the outer raceway groove

$f_i = r_i/D_b$  = Curvature radius coefficient of the inner raceway groove

$r_o$  and  $r_i$  are the Outer and the Inner raceway groove curvature radius, respectively.

## 2.2 Objective Function

In normal operating conditions of rolling element bearings, the main mode of failure is contact fatigue. Therefore, for conventional rolling element bearings the longest fatigue life becomes a basic requirement. Based on the dynamic load rating the objective function [2] can be expressed as

$$\begin{aligned} \text{Maxi}[f(x)] &= \text{Maxi}[f_c Z^{2/3} D_b^{1.8}] & D_b \leq 25.4\text{mm} \\ &= \text{Maxi}[3.647 f_c Z^{2/3} D_b^{1.4}] & D_b > 25.4\text{mm} \end{aligned} \quad (3)$$

where

$$f_c = 37.91 \left\{ 1 + \left[ 1.04 \left( \frac{1-\gamma}{1+\gamma} \right)^{1.72} \left( \frac{f_i(2f_o-1)}{f_o(2f_i-1)} \right)^{0.41} \right]^{10/3} \right\}^{-0.3} \left[ \frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}} \right] \left[ \frac{2f_i}{2f_i-1} \right]^{0.41}$$

Because  $\gamma = D_b \cos \alpha / D_m$  (which appears in the objective function (equation 3)) is not an independent parameter, it does not figure in the list of design parameters. Here  $\alpha$ , the free contact angle depends upon the type of bearing. In the current discussion, the deep groove ball bearing has been considered for which  $\alpha$  is equal to zero.

### 2.3 Constraint Conditions

For the convenience of the bearing assembly, the number and diameter of the balls should satisfy the following requirement [2]

$$2(Z-1) \sin^{-1}(D_b / D_m) \leq \phi_0 \quad (4)$$

Therefore, a constraint condition is

$$g_1(X) = \frac{\phi_0}{2 \sin^{-1}(D_b / D_m)} - Z + 1 \geq 0 \quad (5)$$

where,  $\phi_0$  is the maximum tolerable assembly angle (see Figure 3) which depends upon the bearing geometry.

The diameter of balls should be within a certain range given by

$$K_{D_{\min}} \frac{D-d}{2} \leq D_b \leq K_{D_{\max}} \frac{D-d}{2} \quad (6)$$

where,  $K_{D_{\min}}$  and  $K_{D_{\max}}$  are the minimum and the maximum values of the ball diameter constants respectively, that are connected with the diametric series of bearings and ball strength.

The corresponding constraint conditions are given by

$$g_2(X) = 2D_b - K_{D_{\min}}(D-d) \geq 0 \quad (7)$$

$$g_3(X) = K_{D_{\max}}(D-d) - 2D_b \geq 0 \quad (8)$$

In order to guarantee the running mobility of bearings, the difference between the pitch diameter and the average diameter in a bearing should be less than a certain given value. Therefore,

$$g_4(X) = D_m - (0.5 - e)(D + d) \geq 0 \quad (9)$$

$$g_5(X) = (0.5 + e)(D + d) - D_m \geq 0 \quad (10)$$

where,  $e$  is a constant and is obtained based on the mobility conditions of the balls.

In the design optimization of bearings, a pitch diameter is usually larger than the corresponding average diameter of a bearing. Hence, the thickness of a bearing ring at the outer raceway bottom should not be less than  $\epsilon D_b$ , where  $\epsilon$  is a constant which is obtained from the simple strength consideration of the outer ring. The constraint condition is then

$$g_6(X) = 0.5(D - D_m - D_b) - \epsilon D_b \geq 0 \quad (11)$$

The groove curvature radii of the inner and outer raceways of a bearing should not be less than  $0.515D_b$ . No upper limit is specified for the groove curvature radius, but if it is larger than  $0.52D_b$  and  $0.53D_b$ , respectively for inner and outer raceways, the dynamic load rating of the bearing will decline. Therefore

$$g_7(X) = 0.52 \geq f_i \geq 0.515 \quad (12)$$

$$g_8(X) = 0.53 \geq f_o \geq 0.515 \quad (13)$$

In brief, a design optimization problem is summed up as an optimization of an objective function (equation (3)) and eight inequality constraints expressed by equations (5, 7-13).

### 3. COMPUTATIONAL PROCEDURE

In this section we describe the procedure used for implementation of GAs in solving the given problem.

#### 3.1. Handling Constraints

The method of handling constraints in optimization problems is a critical issue in the domain of GAs. Constraints are usually classified as equality or inequality relations. Since equality constraints may be subsumed into a system model - the black box - the basic consideration is for inequality constraints. GAs generate a sequence of parameters to be tested using the system model, the objective function and the constraints. After running the model the objective function is evaluated and a check is performed to verify whether any constraints are violated. If not, the parameter set is assigned the fitness value corresponding to the objective function evaluation. If constraints are violated, the solution is infeasible and thus has no fitness. This solution works in the current optimization problem given that the domain of feasible solutions is large. Otherwise, if the domain of feasible solutions had been nearly as small as the best solution, then the *penalty technique* [9] would have been required.

#### 3.2 Optimization Procedure

The overall procedure for solving the discrete optimization problem mentioned using GAs is illustrated in Figure 4. The parameters of the GA are specified in the

beginning and an initial population is generated. Given  $pop$  design points, the values of fitness and the design constraints can be calculated for every design point in the current population. The solutions violating any of the constraints are given a fitness value of zero. A new population is created by *selection*, *crossover* and *mutation*. The design variables viz. pitch diameter  $D_m$ , ball diameter  $D_b$ , number of balls  $Z$ ,  $f_i$  and  $f_o$  are varied at this stage. The whole process is repeated until the number of generations reaches the maximum number specified. Since superior individuals in the population have higher probabilities of survival to the next generation, the procedure converges to the best design point after  $maxgen$  generations. The design point normally represents the global optimum, owing to the evolutionary nature of the algorithm.

## 4. ENCODING THE PROBLEM

Representing the problem to suit the algorithm is a crucial issue. The following has to be performed before starting off the GA.

### 4.1 Coding

In order to use GAs to solve problems, variables  $x_i$ 's of the function  $f(x)$  are first coded in some string structures. Generally binary-coded strings having 1's and 0's are used. The length of the string is usually determined depending on the desired accuracy of the solutions. For instance, if four bits are used to code each variable in a two-variable

function optimization problem, the strings (00000000) and (11111111) would represent the extreme points respectively, because the sub strings (0000) and (1111) have the minimum and the maximum decoded values.

In the problem at hand, decimal mapping has been used and the solution has the least significant bit of the order of  $10^{-6}$  Newton, which is sufficient for the given problem. Since there are five parameters there are five sub-strings each of length five digits. Each point  $x$  can be denoted by  $x = (x_1, x_2, x_3, x_4, x_5)$ . The function value at the point  $x$  can be calculated by substituting  $x$  in the given objective function  $f(x)$ . The strings are generated in such a way that they do not violate any constraints. This ensures that the working space is valid.

## **4.2 Fitness Function**

As pointed out earlier, GAs mimic the survival-of-the-fittest principle of nature to make a search process. In general, a *fitness* function  $F(x)$  is first derived from the objective function and used in successive genetic operations. In the current case, which is a maximization problem, the fitness function can be considered to be the same as the objective function. The fitness function value of a string is known as the string's *fitness*.

The operation of GAs begins with a population of random strings representing design or decision variables. Thereafter, each string is evaluated to find the fitness value. The population is then operated by three main genetic operations - *reproduction*,

*crossover*, and *mutation* - to create a new population of points. The new population is further evaluated and tested for termination. If the termination criterion (the maximum number of iterations specified by the user) is not met, the population is iteratively operated by the above three operators and evaluated. This procedure is continued till the termination criterion is met. One cycle of these operations and the subsequent evaluation procedure is known as a *generation* in GA terminology.

## 5. RESULTS AND CONCLUSIONS

Classical optimization approaches usually consider only three design parameters viz. pitch diameter,  $D_m$ , ball diameter,  $D_b$  and the number of balls,  $Z$ , for reasons of complexity [2, 13]. In the present analysis we have used five design parameters (viz.  $D_m$ ,  $D_b$ ,  $Z$ ,  $f_i$  and  $f_o$ ). The best solutions obtained from the present analysis of the given problem are provided in Table 1 for different sets of boundary dimensions i.e. the outer diameter,  $D$  and the bore,  $d$ . In the constraints, the following values have been used for the constants:  $K_{Dmin} = 0.5$ ,  $K_{Dmax} = 0.8$ ,  $e = 0.1$ ,  $\epsilon = 0.1$  and  $\varphi_o = 4.7124$  rads, which are obtained from considerations of the mobility of balls and strength of the ball and rings. The GA parameters used are: Population Size = 300, Cross-Over Probability = 0.5 and Mutation Probability = 0.15.

Tables 2 (a), (b) and (c) throw more light on the convergence and the fact that the solution is most likely an optimal one and not one trapped in local maxima. Table 2(a)

shows that even with an increase in mutation probability, the reported values of the dynamic capacity have hardly shown any discernable variation. Mutation facilitates a random jump out of local maxima (or minima). Normally the value of mutation rate is kept low to ensure that the search does not become too random. Even at a high mutation rate of 0.35, the value of  $C$  does not differ much from that reported when the rate is an optimum of 0.05.

Table 2(b) shows solutions arrived at varying values of crossover probability. Here too the dynamic capacities have hardly varied. Hovering around the neighbourhood of 7300, these values are much the same as those reported in Table 2(a). Increasing the number of generations (Table 2(c)) seem to produce better values of dynamic capacity but for higher numbers they seem to saturate to values reported in Tables 2(a) and (b). Of special significance are those reported for population sizes of 5000 and 7000 (Table 2(c)) wherein the results derived are identical, thus clearly indicating high precision.

The dynamic load rating,  $C$ , also referred to as optimum technical index, is used to compare the results of the existing design [26] with those of the present ones (see Table 1). A marked improvement in the value of  $C$  has been observed. While the standard values of  $C$  for most of the sets are low, the ones found using GAs are about one and a half to two times higher. The marginally larger values of  $C$  in these cases may however be an indication as to the global optimality of the solutions.

In order to compare the increase in the fatigue life of the bearings designed using GAs as against those using available standard values, the following relation has been used

$$\lambda = L_g / L_s = (C_g / C_s)^3 \quad (14)$$

This has been derived from equation (1) using a constant applied load  $F$  to both, the bearings designed using GAs and those designed using available values. The subscripts  $g$  and  $s$  represent the values computed using GAs and those currently available, respectively. From Table 1 it can be observed that there is a quantum jump in the fatigue life of the bearings designed using the GA approach as compared to the ones designed using the standard values, the order of increase in the life ranging from four to eight.

It can thus be clearly seen that the computed values of dynamic load rating  $C$  and the increase in the fatigue life are far better than those reported in standard catalogues [15].

The computation time taken is far lower in the current method, a feature that adds further credibility to the use of GAs in the design. The implementation of the program was done using C running on a Pentium II 500MHz Personal Computer.

The software developed for the design of the rolling element bearings thus serves as a tool to generate the values of the dynamic load rating  $C$  and  $D_b, D_m, f_i, f_o, Z$ , parameters that define the internal geometry, given various values of boundary dimensions ( $D, d$ ).

From the manufacturing perspective, the knowledge of the internal geometry could be used in analyzing the bearing which gives insights into the stresses that the bearings can sustain, the deformations that may occur, kinematical issues and other such performance characteristics even before the actual fabrication of the bearings.

The currently available sources for bearing design provide only the boundary dimensions and performance characteristics. Conventional optimization methods fail to incorporate so many design variables (five used in the present work) and do not take care of the inherent local maxima (or minima) problems. Another advantage of using the GA approach is that the number of design variables can be easily increased, if required, without much computational efforts.

With hardly any source that can provide these values, manufacturers as well as prospective bearing designers will find this tool to be efficient and a highly productive one. Furthermore the corresponding values of  $C$  serve as a measure of the fatigue life of the bearings.

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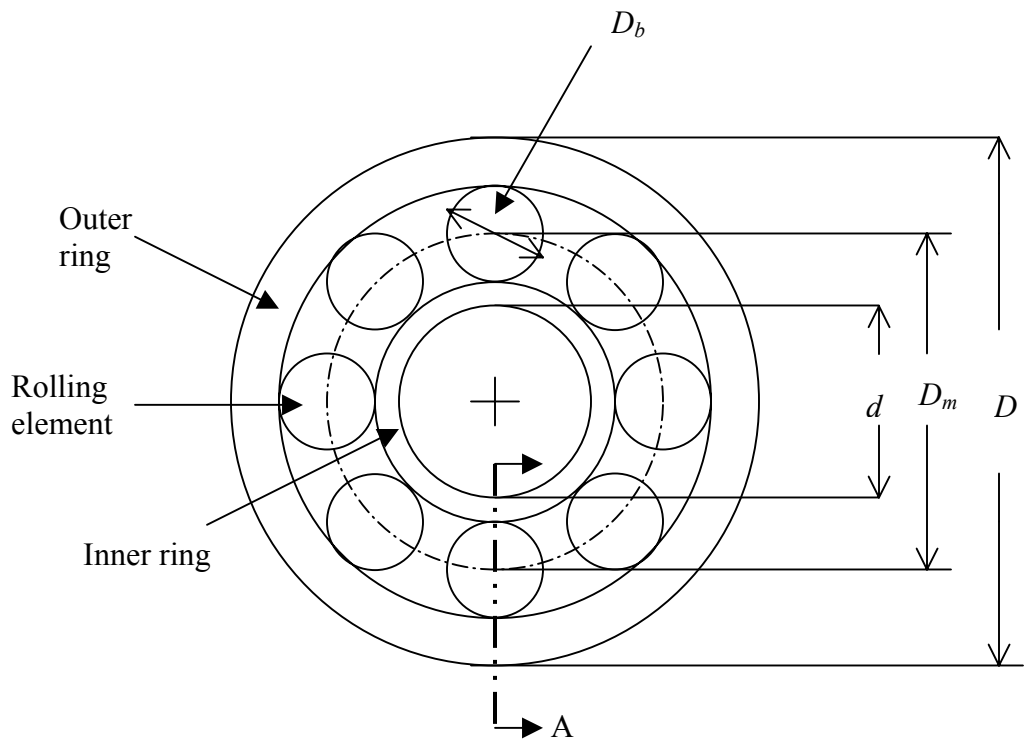


Figure 1. Rolling Element Bearing macro-geometry.

Bearing rotational axis

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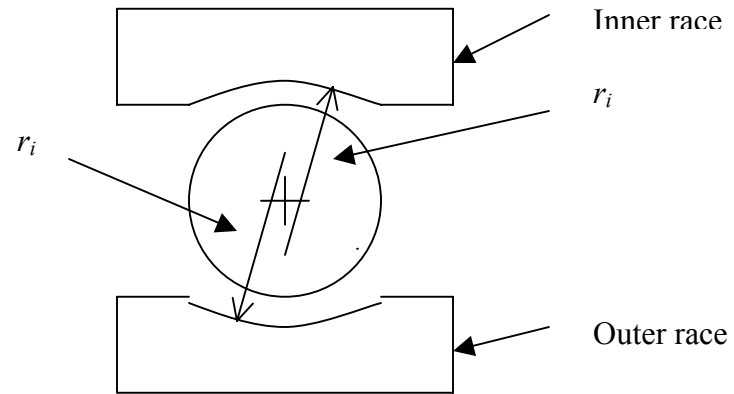


Figure 2. Cut sections of Bearing Races (Sectional plane A, see Figure 2)

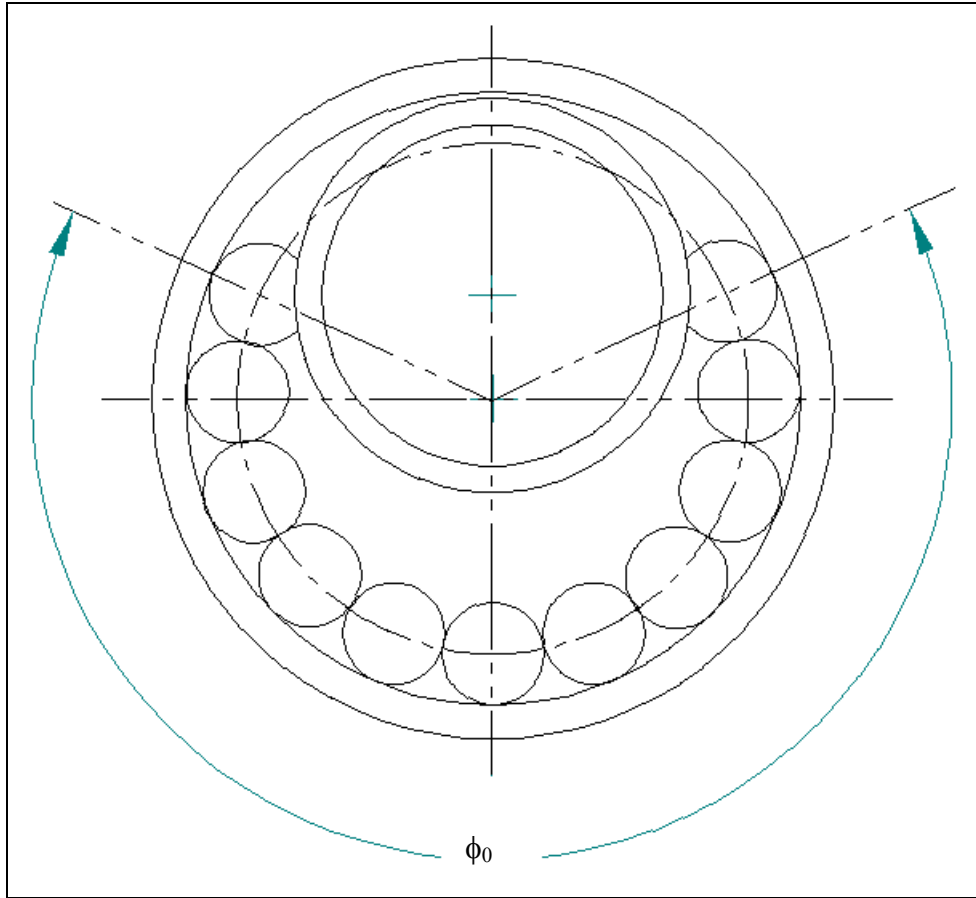


Figure 3. Ball bearing showing the assembly angle

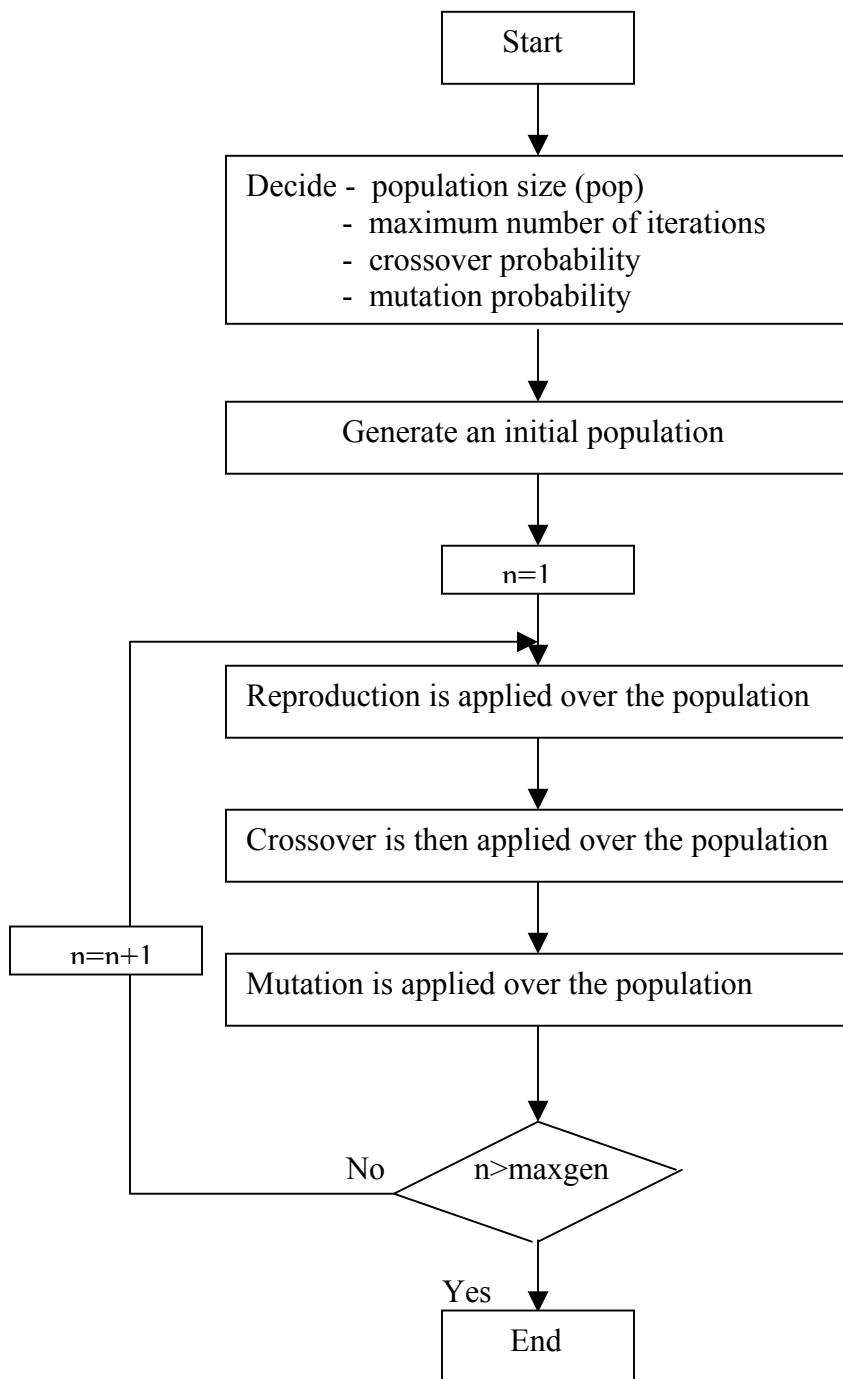


Figure 4. Flowchart for the solution of a problem using GA

Table 1 Optimum design parameters of rolling element bearings through GA  
(Population Size = 300, Cross-Over Probability = 0.5 & Mutation Probability = 0.15)

Boundary dimensions (Input)		Design variables (Output)					Performance comparisons (Calculations are based on the output)		
$D$ (mm)	$D$ (mm)	$D_b$ (mm)	$D_m$ (mm)	$f_i$	$f_o$	$Z$	$C_g$ (Newton)	$C_s$ (Newton) [26]	$\lambda$
30	10	7.4705	19.5321	0.5150	0.5150	7	7306.6	3580	8.50
35	15	7.9019	23.9355	0.5150	0.5150	8	9553.7	5870	4.31
47	20	10.5968	32.1626	0.5150	0.5152	8	16213.4	9430	5.08
62	30	12.7942	44.0785	0.5150	0.5150	9	25785.0	14900	5.18
80	40	15.9971	57.5179	0.5150	0.5157	9	38979.1	22500	5.20
90	50	15.8214	67.8040	0.5150	0.5150	11	45161.4	26900	4.73
110	60	19.3331	82.9124	0.5152	0.5152	11	64542.3	40300	4.11
125	70	21.9887	94.1924	0.5150	0.5150	11	81701.2	47600	5.06
140	80	23.9952	105.8926	0.5150	0.5150	11	95915.9	55600	5.13
160	90	27.9955	120.7310	0.5150	0.5151	11	121401.9	73900	4.43

Table 2. Variation of best solution (for  $D = 30$  mm  $d = 10$  mm)  
with respect to GA parameters

2(a) Population Size = 300, Crossover probability = 0.5, Number of Generation = 10000

Mutation Probability	Dynamic Capacity, $C_g$
0.01	7297.4
0.05	7305.9
0.10	7295.6
0.15	7286.9
0.20	7302.6
0.25	7307.5
0.30	7285.1
0.35	7296.7

2(b) Population Size=300, Mutation probability = 0.15, Number of Generation =10000

Crossover probability	Dynamic Capacity, $C_g$
0.1	7305.3
0.2	7305.3
0.3	7308.0
0.4	7291.2
0.5	7286.9
0.6	7300.0

2(c) Population Size = 300, Mutation Probability= 0.15, Crossover Probability = 0.5

Number of Generations	Dynamic Capacity, $C_g$
100	7191.7
200	7286.6
300	7286.6
400	7286.6
500	7286.6
700	7286.6
1000	7286.6
5000	7302.8
7000	7302.8