The Role of Payout Horizon in Determining the Risks and Returns on Claims to Aggregate Cash Flows

Motivation

- Systematic variation in *forecasts* of cash flow growth – i.e. 'cash flow risks' – are increasingly recognized as important for asset pricing.

- for the market's cash flows: Bansal, Yaron (2004, 2006)

- at the level of firm and portfolio cash flows: Campbell, Vuolteenaho (2004), Bansal, Dittmar, Lundblad (2005), Lettau, Wachter (2006), Kiku (2006), Bansal, Dittmar, Kiku (2006), Croce, Lettau, Ludvigson (2006).

Motivation

- The importance of cash flow forecasts motivates the following:

  Question: How does the payout/forecast horizon of a cash flow claim affect its asset pricing properties?

  - its sensitivity to 'cash-flow' news **and** 'discount-rate' news
  - return volatility, CAPM beta, risk premia, CAPM alpha, etc . . .

  Question: how does payout horizon matter at the level of firms and portfolios? (i.e. as in Lettau and Wachter (2006))

Forecasting: Challenges of 2 Long Run Considerations

1. *Long run risk* in consumption-dividend growth

   - intuitively $\Rightarrow$ uncertainty/risk accumulates with payout horizon

   - *Moreover* the corporate sector is extra-sensitive/levered to consumption growth (BY 2004)

2. *Cointegration*: Aggregate dividends share of consumption is (covariance) stationary.

   - intuitively $\Rightarrow$ at long enough horizons, aggregate dividend and consumption payouts have approx. the same risk

Intuition

- Cash flow "news" changes investors' dividend forecasts mostly at short and intermediate horizons.

- Cointegration "bounds" long run risk

- Result: The interaction of long run risk and cointegration causes exposures to aggregate risks to *rise* sharply in the short to intermediate horizon and then *decrease* towards a long-run level. In the 'knife-edge' case where cointegration is *eliminated*, risk *increases* monotonically in the payout horizon.

Overview

1. Solve an equilibrium model with long run risks and cointegrated aggregate dividends and consumption.

2. Solve for asset pricing affects at each payout horizon.

3. Can horizon induce a cross-section of expected returns? Can the model establish a 'value-premium'?

4. Examine why the CAPM succeeds in the model. Compare to results in Campbell-Vuolteenaho (2004) and Lettau-Wachter(2006)

   - look at discount rate risk and cash flow risk at each horizon

   - the drivers of risk premia and volatility/beta at each horizon

The Model

Bansal and Yaron (2006) general equilibrium setting:

1. Rep agent with Epstein-Zin preferences

2. γ is the coefficient of relative risk aversion

3. ψ is the IES parameter

The Model

$$\Delta c_{t+1} = g_{t+1} = \mu_c + x_t + \sigma_t \eta_t \quad (1)$$
$$x_{t+1} = \rho x_t + \sigma_t \varphi_e e_{t+1} \quad (2)$$
$$\sigma^2_{t+1} = \sigma^2 + \nu(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \quad (3)$$
$$d_{t+1} - c_{t+1} \equiv \mu_{d-c} + s_{t+1} \quad (4)$$
$$s_{t+1} = \rho_s s_t + \phi_{sx} x_t + \varphi_d \sigma_t u_{t+1} \quad (5)$$

and $\mathrm{v}_{t+1} = (\eta_{t+1}, e_{t+1}, w_{t+1}, u_{t+1})' \overset{\text{iid}}{\sim} \mathcal{N}(0, \mathbf{\Omega})$ with

$$\mathbf{\Omega} = \begin{pmatrix} 1 & \rho_{\eta e} & 0 & \rho_{\eta u} \\ \rho_{\eta e} & 1 & 0 & \rho_{eu} \\ 0 & 0 & 1 & 0 \\ \rho_{\eta u} & \rho_{eu} & 0 & 1 \end{pmatrix}$$

$$\Delta d_{t+1} \equiv \Delta s_{t+1} + \Delta c_{t+1} \Rightarrow$$

$$\Delta d_{t+1} = \mu_c + (1 + \phi_{sx}) x_t + (\rho_s - 1) s_t + \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1}$$

The Market

- Claim to aggregate dividend payouts net of issuance

- $z_{m,t}$ is the log price-dividend ratio

- $r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}$

- $z_{m,t} = A_{0,d} + A_{1,d} x_t + A_{2,d} \sigma_t^2 + A_{3,d} s_t$
  - s_t is a state variable since it governs Δd_{t+1}

$$A_{3,d} = \frac{\rho_s - 1}{1 - \kappa_{1,m}\rho_s}$$

$$\Delta d_{t+1} = \mu_c + (1 + \phi_{sx})x_t + (\rho_s - 1)s_t + \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1}$$

- $A_{3,d} < 0$ since $s_t > 0$ implies lower expected dividend growth

- $A_{3,d}$ can be interpreted as the 'ex-dividend' value of a unit *increase* in dividends (with consumption unchanged)

$$(\rho_s - 1) + \kappa_{1,m}(\rho_s - 1)\rho_s + \kappa_{1,m}^2(\rho_s - 1)\rho_s^2 + \cdots = (\rho_s - 1)\sum_{n=0}^{\infty}(\kappa_{1,m}\rho_s)^n = A_{3,d}$$

- the *'cum-dividend'* value is $1 + \kappa_{1,m}A_{3,d} = \dfrac{1 - \kappa_{1,m}}{1 - \kappa_{1,m}\rho_s}$

$$A_{1,d} = \frac{1 - \frac{1}{\psi} + \phi_{sx}\left(\frac{1-\kappa_{1,m}}{1-\kappa_{1,m}\rho_s}\right)}{1-\kappa_{1,m}\rho}$$

$$\Delta d_{t+1} = \mu_c + (1 + \phi_{sx})x_t + (\rho_s - 1)s_t + \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1}$$

- if $x_t \uparrow$ by 1 then 'levered' div. growth $\uparrow$ by ϕ_{sx} today, $\phi_{sx} \times \rho^1$ in 1 period, . . . , $\phi_{sx} \times \rho^j$ in j periods, . . .

- present value is: $\sum_{j=0}^{\infty} \phi_{sx} \times \rho^j \times \frac{1-\kappa_{1,m}}{1-\kappa_{1,m}\rho_s} \times \kappa_{1,m}^j = \frac{\phi_{sx}(\frac{1-\kappa_{1,m}}{1-\kappa_{1,m}\rho_s})}{1-\kappa_{1,m}\rho}$

- $\frac{1-\frac{1}{\psi}}{1-\kappa_{1,m}\rho}$ term is due to increase in non-levered dividend growth

Cointegration and the Market Return

$$r_{m,t+1} - E_t(r_{m,t+1}) =$$
$$= \sigma_t \eta_{t+1} + \kappa_{1,m} A_{1,d} \varphi_e \sigma_t e_{t+1} + \kappa_{1,m} A_{2,d} \sigma_w w_{t+1} + \underbrace{(\kappa_{1,m} A_{3,d} + 1)}_{\text{cum-dividend value}} \varphi_d \sigma_t u_{t+1}$$

- 'cum-dividend' value determines exposure to volatile u_{t+1} innovation

- sensitivity to ρ_s value: $(\kappa_{1,m} A_{3,d} + 1) = \frac{1-\kappa_{1,m}}{1-\kappa_{1,m}\rho_s} = \frac{\frac{1}{\kappa_{1,m}}-1}{\frac{1}{\kappa_{1,m}}-\rho_s}$

$\rho_s = 1$ (no cointegration) $\Rightarrow$ exposure $= 1$

$\rho_s = 0.995$ ('loose' cointegration) $\Rightarrow$ exposure ≈ 0.5

(note: $\frac{1}{\kappa_{1,m}} \approx 1.005$)

Data and Model Output

	Data: Bansal-Yaron(2006)		Model
	Estimate	Std. Error	
$E[\Delta c]$	1.96	0.32	1.94
$\sigma(\Delta c)$	2.20	0.45	2.39
$E[\Delta d - \chi]$	0.76	1.47	1.23
$\sigma(\Delta d - \chi)$	23.11	3.54	23.03
$E[R_m - R_f]$	7.62	1.86	7.01
$\sigma(R_m)$	19.9	2.52	16.02
$E[R_f]$	0.85	0.40	1.65
$\sigma(R_f)$	1.22	0.33	0.95
$E[v - d]$	3.04	0.09	2.88
$\sigma(v - d)$	0.34	0.04	0.39

The data is taken directly from Table 1 and Table 10 in Bansal and Yaron (2006). The Model Output is for parametrization I: $\rho_s = 0.994$, $\rho = 0.980$, $\sigma = 0.0048$, $\phi_{sx} = 4.8$, $\varphi_e = 0.037$, $\varphi_d = 10.5$, $\nu = 0.983$, $\sigma_w = 7e - 006$, $\gamma = 12$, $\psi = 1.5$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.35$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

Model Output: Parametrization II ($\rho_s = 1$)

	Param I ($\rho_s = 0.994$)	Param II ($\rho_s = 1$)
$E[\Delta c]$	1.94	1.94
$\sigma(\Delta \mathrm{c})$	2.39	2.39
$E[\Delta d - \chi]$	1.23	1.04
$\sigma(\Delta d - \chi)$	23.03	23.60
$E[R_m - R_f]$	7.01	12.13
$\sigma(R_m)$	16.02	34.70
$E[R_f]$	1.65	1.65
$\sigma(R_f)$	0.95	0.95
$E[v - d]$	2.88	2.59
$\sigma(v - d)$	0.39	0.29

Model Output for parametrization II: $\rho_s = 1$, $\rho = 0.980$, $\sigma = 0.0048$, $\phi_{sx} = 4.8$, $\varphi_e = 0.037$, $\varphi_d = 10.5$, $\nu = 0.983$, $\sigma_w = 7e - 006$, $\gamma = 12$, $\psi = 1.5$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.35$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

Horizon Analysis with Dividend Strips

- "Aggregate Dividend Strip": A Claim to the aggregate dividend payout at a fixed future time, called its *maturity*.
- market is the sum of all the strips
- central to models in Lettau and Wachter (2006), Croce, Lettau, and Ludvigson (2006).
  - they model a 'firm' as a portfolio of aggregate dividend strips
  - strips are quite a flexible modeling tool
- We price the strips and examine the relationship between their maturity and asset pricing properties

Solving for Strip Prices

- $P_{n,t}$: time t price of a strip that matures in n periods.

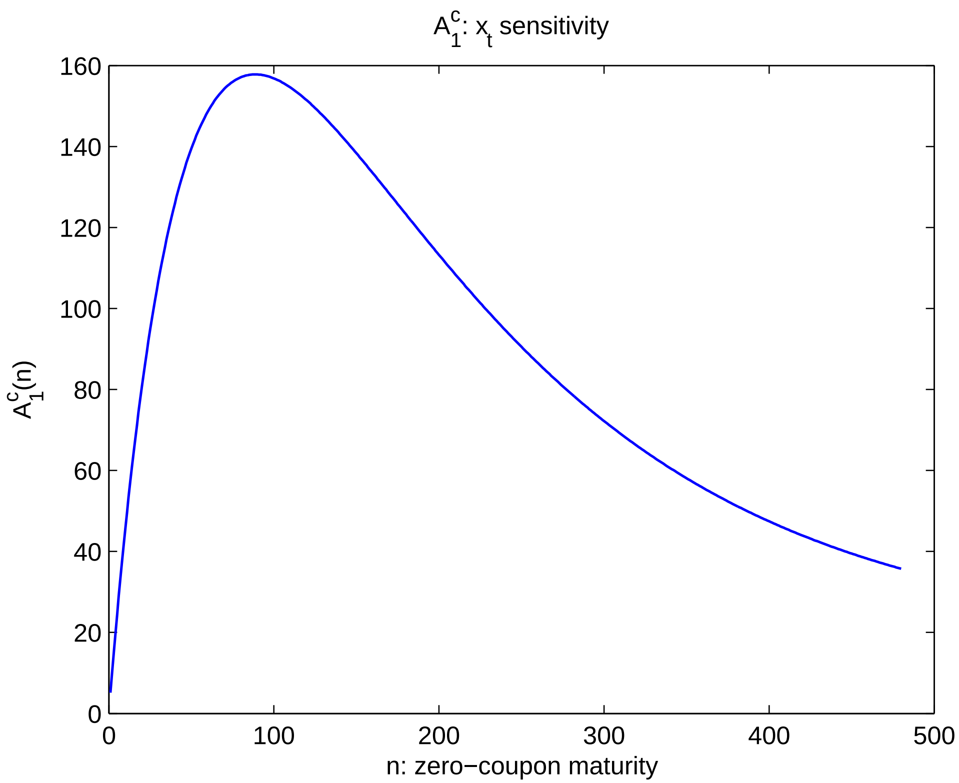
- $P_{n,t} = E_t \left[M_{t+1} P_{n-1,t+1} \right] \Rightarrow \frac{P_{n,t}}{D_t} = E_t \left[M_{t+1} \frac{P_{n-1,t+1}}{D_{t+1}} \frac{D_{t+1}}{D_t} \right]$

- conjecture $\frac{P_{n,t}}{D_t} = \exp\left(A_0^c(n) + A_1^c(n) x_t + A_2^c(n) \sigma_t^2 + A_3^c(n) s_t \right)$

  - boundary condition: $\frac{P_{0,t}}{D_t} = 1$

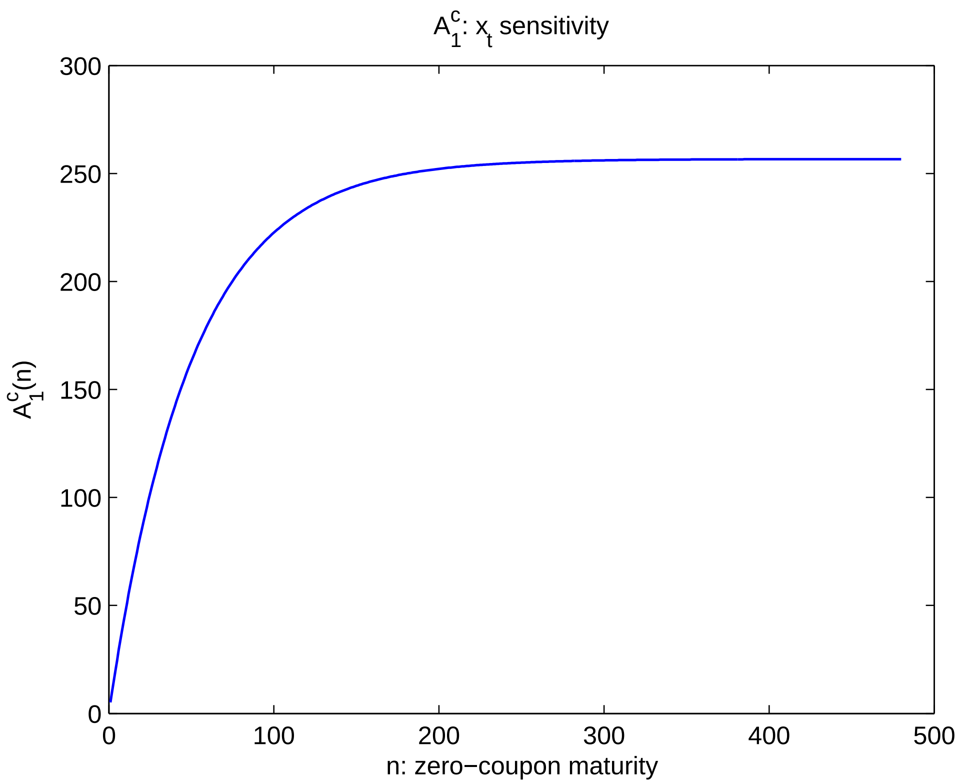
- $\Rightarrow$ recursive solution of $A_i^c(n)$ and $A_0^c(0) = A_1^c(0) = A_2^c(0) = A_3^c(0) = 0$

$A_1^c(n)$ vs. n for parametrization I

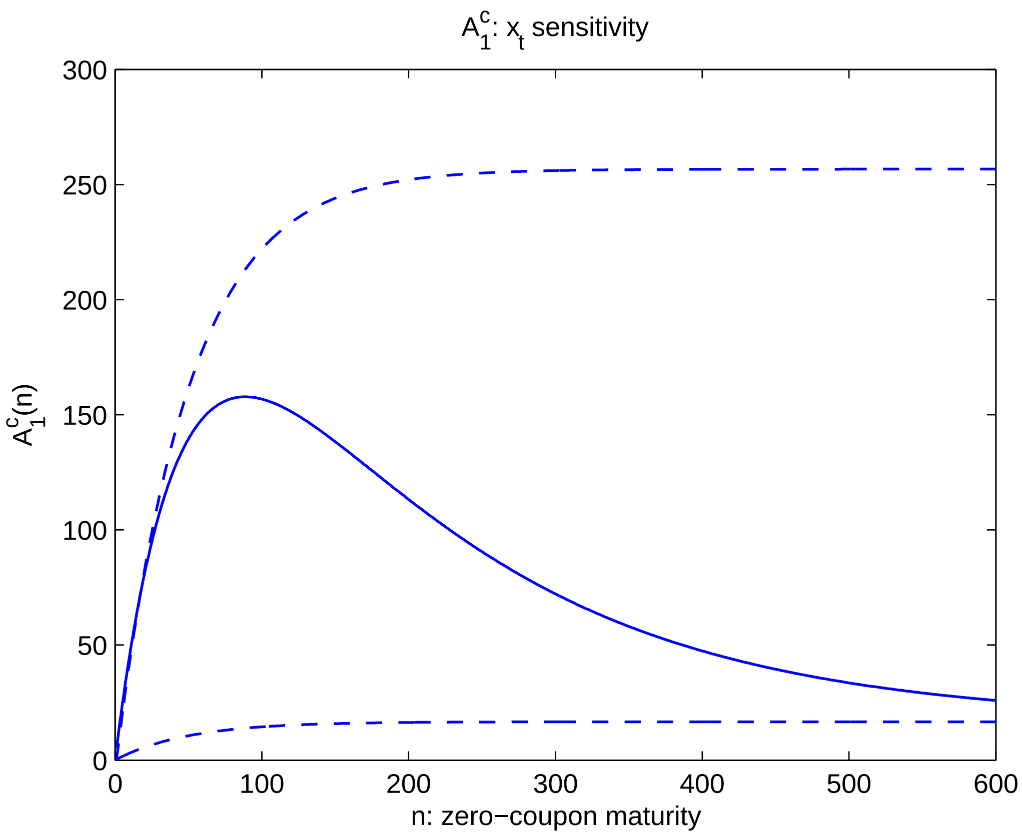


$\rho_s = 0.994$, $\rho = 0.980$, $\sigma = 0.0048$, $\phi_{sx} = 4.8$, $\varphi_e = 0.037$, $\varphi_d = 10.5$, $\nu = 0.983$, $\sigma_w = 7e-006$, $\gamma = 12$, $\psi = 1.5$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.35$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

$A_1^c(n)$ vs. n for parametrization II $(\rho_s = 1)$

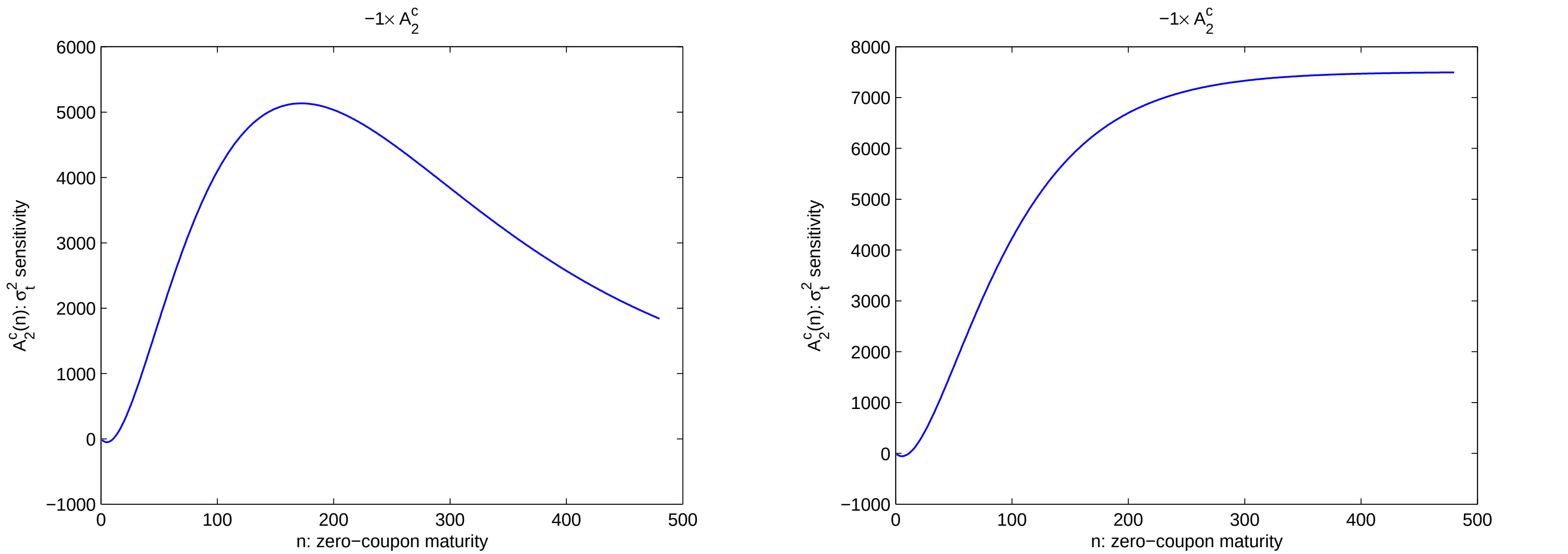


$\rho_s = 1$, $\rho = 0.980$, $\sigma = 0.0048$, $\phi_{sx} = 4.8$, $\varphi_e = 0.037$, $\varphi_d = 10.5$, $\nu = 0.983$, $\sigma_w = 7e-006$, $\gamma = 12$, $\psi = 1.5$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.35$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$



The figure displays a comparison of term structures of A_1^c. The sold line is for parametrization I. The dashed line forming the upper bound is for parametrization I with $\rho_s = 1$ (no cointegration). The lower bound is for a 'consumption-like' dividend stream where $\phi_{sx} = 0$.

$-1 \times A_2^c(n)$ vs. n for parameterizations I and II



Notes: The figure displays the term structure of A_2^c, the sensitivity of zero-coupon equity to σ_t^2, under the equilibria of parametrization I (on the left) and II (on the right).

Fun Facts: A Connection with the Market

$$r_{m,t+1} - E_t(r_{m,t+1}) = \sigma_t \eta_{t+1} + \underline{\kappa_{1,m} A_{1,d}} \varphi_e \sigma_t e_{t+1} + \underline{\kappa_{1,m} A_{2,d}} \sigma_w w_{t+1} + \underline{(\kappa_{1,m} A_{3,d} + 1)} \varphi_d \sigma_t u_{t+1}$$

$$r^n_{c,t+1} - E_t(r^n_{c,t+1}) = \sigma_t \eta_{t+1} + \underline{A^c_1(n-1)} \sigma_t \varphi_e e_{t+1} + \underline{A^c_2(n-1)} \sigma_w w_{t+1} + \underline{(1 + A^c_3(n-1))} \varphi_d \sigma_t u_{t+1}$$

- $1 + \kappa_{1,m} A_{3,d}$ is the value-weighted average of $1 + A^c_3(n-1)$: with $\rho_s = 0.99$, $\kappa_{1,m} = 0.996$, market's exposure to u_{t+1} is same as 123 month strip's (i.e. market has a 123 month "duration")

- Letting $\rho_s \to 1$ (the *no* cointegration case), market's exposure to u_{t+1} is same as $\dfrac{\kappa_{1,m}}{1-\kappa_{1,m}} \approx 249$ month strip's

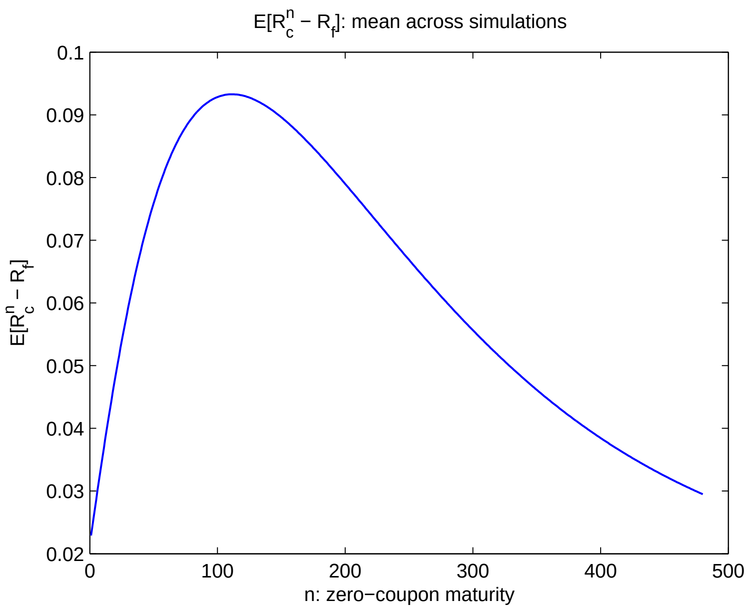
Risk Premia by Horizon

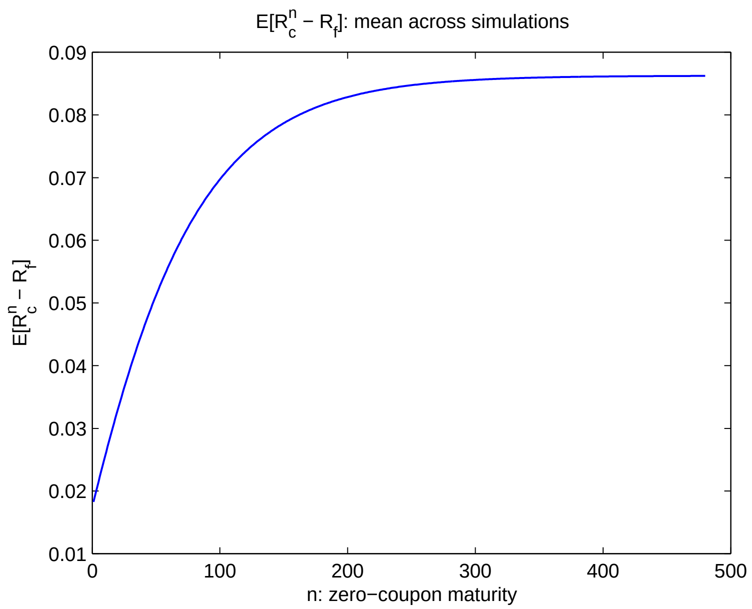
- Since risks/sensitivities vary a lot with horizon, what about risk premia?

- Add the array of strip prices to the the model simulation

- Obtain the term structure of strips' annual risk premia (unconditional)

(annual) Excess Return vs. maturity for parameterizations I and III





The figure displays the term structure of (unconditional) excess returns for the set of zero coupon strips. The excess returns are taken from the **simulation** of the economy under parametrization I (left) and III (right). The simulation assumes a monthly decision interval and output is aggregated to an annual frequency.

Connection to Firms

- What does the previous figure imply about the relationship between a firm's payout horizon (duration) and its expected return?

- firms overweight in *'intermediate term'* strips have the *highest* risk premia

- firms overweight in *long term* strips have the *lowest* risk

- horizon can contribute to cross-sectional differences in expected returns

A ‘Value’ Premium’?

- Could horizon differences help explain the value premium?

- i.e. if we add firms to the model:
  - long-horizon (‘growth’?) firms would have lowest premium (as in LW)
  - intermediate-horizon (’value’?) firms would have highest premium

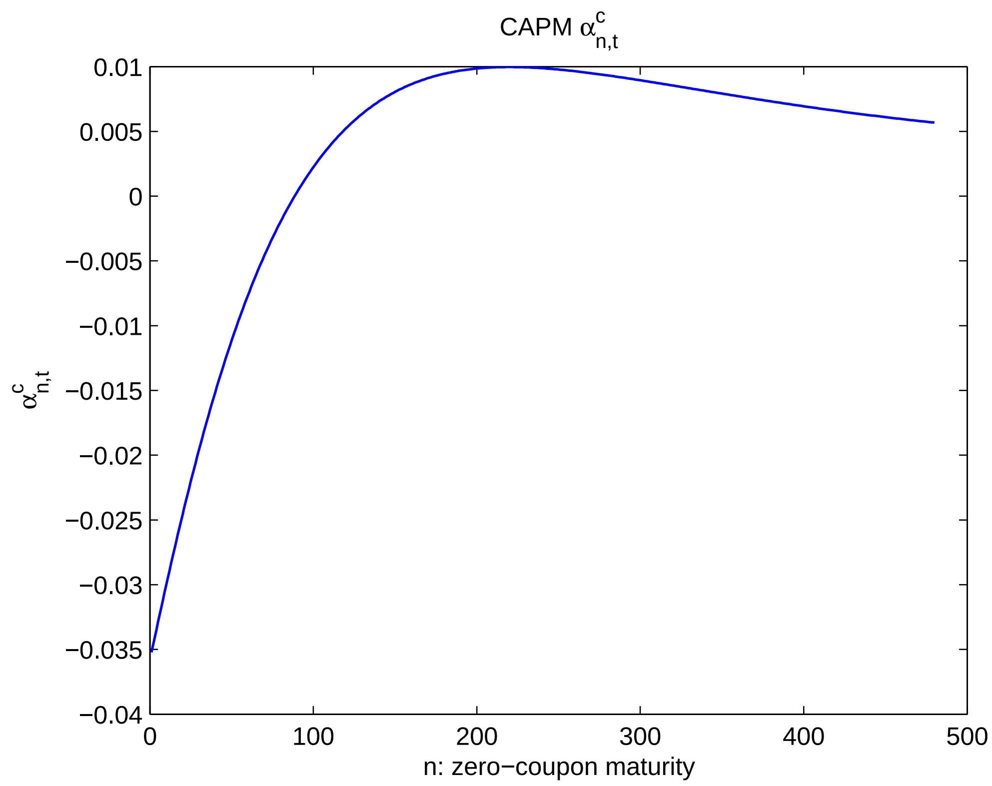
The CAPM

- *However*, under the model's calibration, the CAPM works well on the cross section of dividend strips.

$\Rightarrow$ The CAPM will also work well for portfolios of dividend strips, i.e. for firms.

- so CAPM alphas will be small

CAPM alpha vs. maturity for parametrization I



$\rho_s = 0.994$, $\rho = 0.980$, $\sigma = 0.0048$, $\phi_{sx} = 4.8$, $\varphi_e = 0.037$, $\varphi_d = 10.5$, $\nu = 0.983$, $\sigma_w = 7e-006$, $\gamma = 12$, $\psi = 1.5$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.35$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

Why does the CAPM work in the model?

1. to answer: decompose strips' risk premia and market covariation into parts, each part corresponding to one of A_i^c

   – A_1^c part is *cash flow risk*, A_2^c is *discount rate risk*

   – how important is each risk for premia? for volatility/beta?

2. How do the cash flow and discount rate risks here compare to CV(2004) or LW (2006), two models where CAPM fails?

3. Can this equilibrium model achieve a CV (2004)-style CAPM failure, based on CV's distinct 'roles' for cash flow and discount rate risk?

Risk Premia Decomposed by Risk Source

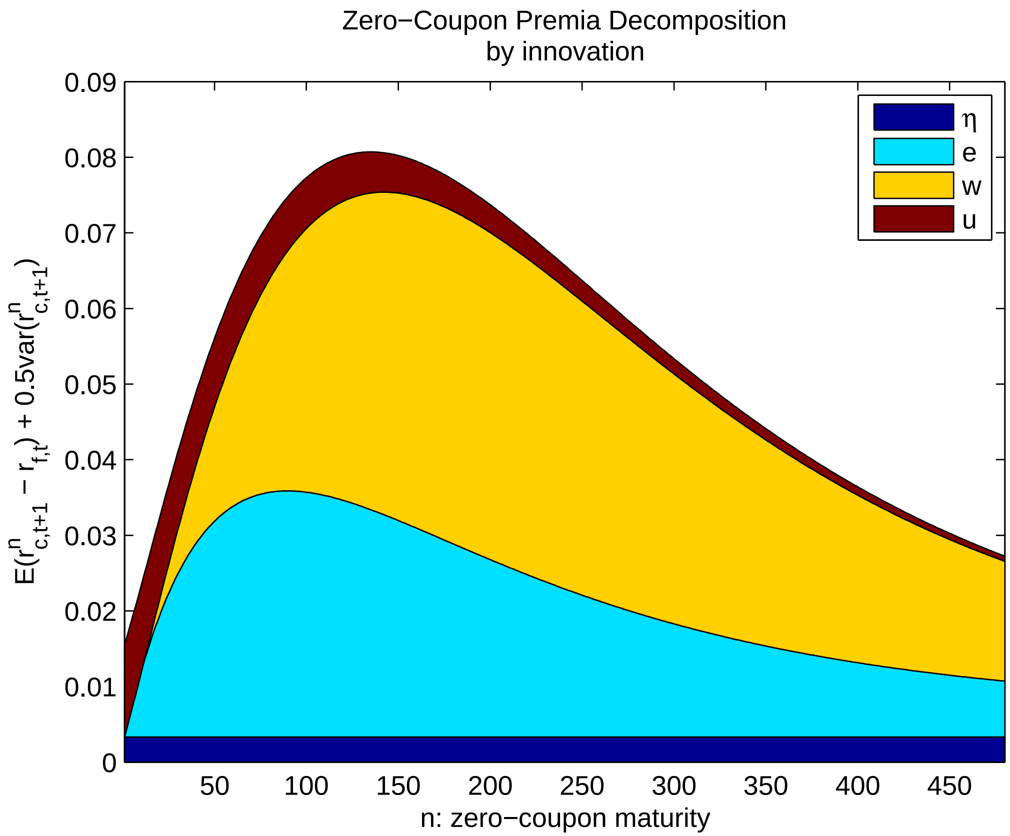
- from the $A_i^c(n)$ we get strips' risk premia *for each horizon*

$$E_t(r_{c,t+1}^n - r_{f,t}) + \frac{1}{2}\text{var}_t(r_{c,t+1}) = \underbrace{(\gamma + \lambda_{m,e}\rho_{\eta e})\sigma_t^2}_{\eta \text{ term}} + \underbrace{A_1^c(n-1)\varphi_e(\gamma\rho_{\eta e} + \lambda_{m,e})\sigma_t^2}_{e \text{ term}}$$
$$+ \underbrace{A_2^c(n-1)\lambda_{m,w}\sigma_w^2}_{w \text{ term}} + \underbrace{\left(1 + A_3^c(n-1)\right)\varphi_d(\gamma\rho_{\eta u} + \lambda_{m,e}\rho_{eu})\sigma_t^2}_{u \text{ term}}$$

- risk premia = sum over the four innovations (η, e, w, u), of a strip's *exposure* to an innovation (its A_i^c coefficient) weighted by the *total compensation* for that risk

- e and w innovations are the model's main sources of *cash flow risk* and *discount rate risk*, respectively

Decomposition of conditional risk premia by innovation sensitivity: parametrization I



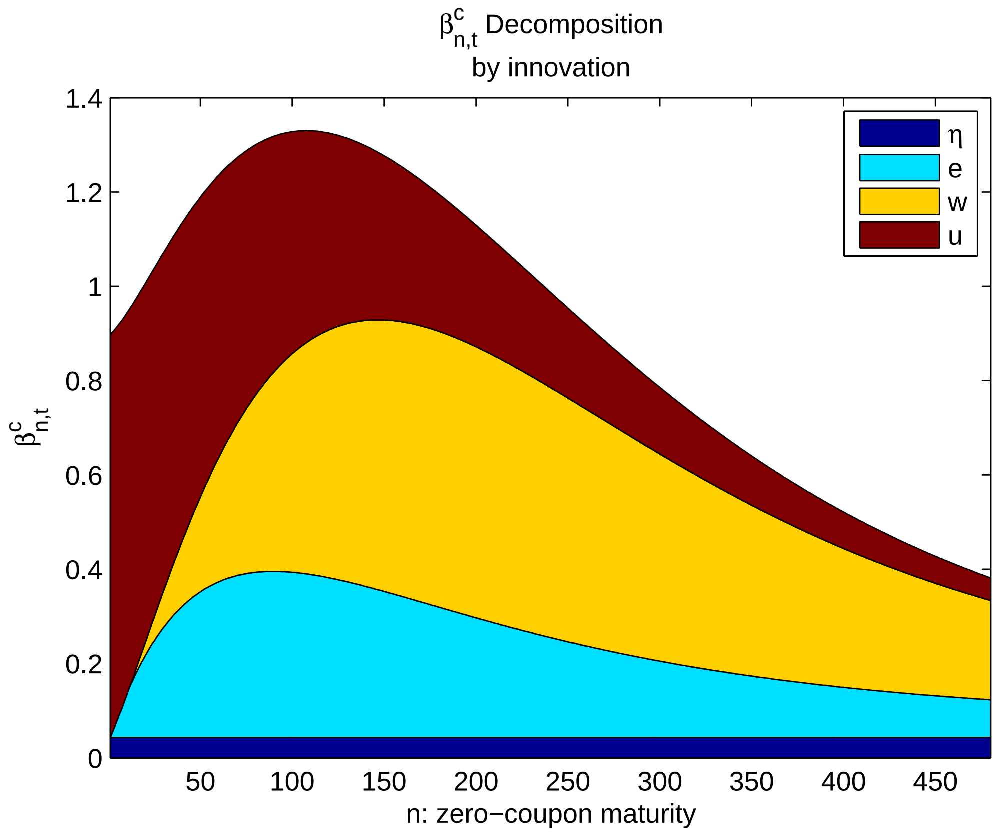
e risk (blue) = *cash flow risk*
w risk (orange) = *discount rate risk*

Betas and Alphas Decomposed by Risk

$$\text{cov}_t(r^n_{c,t+1} - E_t(r^n_{c,t+1}), r_{m,t+1} - E_t(r_{m,t+1})) =$$
$$\underbrace{\big(1 + \beta_{m,e}\rho_{\eta e} + \beta_{m,u}\varphi_d\rho_{\eta u}\big)\sigma_t^2}_{\eta \text{ contribution}} + \underbrace{A_1^c(n-1)\varphi_e\big(\rho_{\eta e} + \beta_{m,e} + \beta_{m,u}\varphi_d\rho_{eu}\big)\sigma_t^2}_{e \text{ contribution}}$$
$$+ \underbrace{A_2^c(n-1)\beta_{m,w}\sigma_w^2}_{w \text{ contribution}} + \underbrace{\big(1 + A_3^c(n-1)\big)\varphi_d\big(\rho_{\eta u} + \beta_{m,e}\rho_{eu} + \beta_{m,u}\varphi_d\big)\sigma_t^2}_{u \text{ contribution}}$$

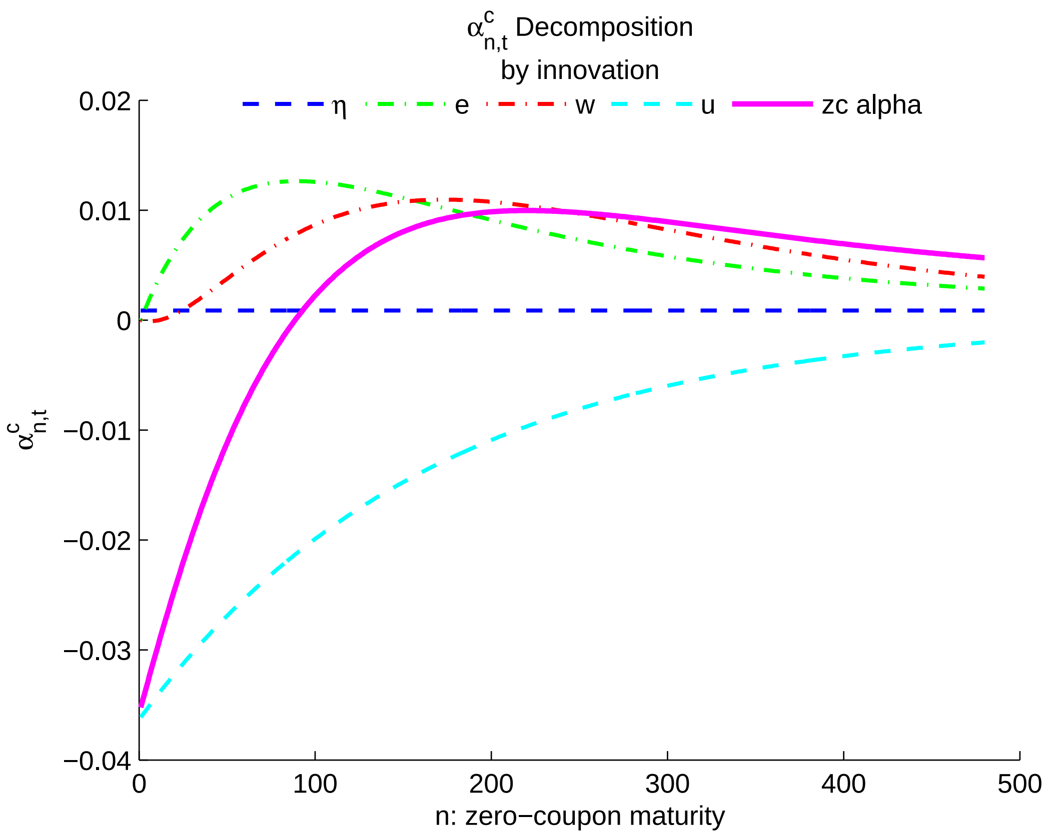
1. shows contribution of each innovation/risk to a strips' market covariance
2. beta decomp = cov decomp / $\text{var}_t(r_{m,t+1})$
3. alpha decomp = risk premia decomp - beta decomp $\times$ market premium

Decomposition of conditional CAPM betas by innovation sensitivity: parametrization I



e risk (blue) = *cash flow risk*
w risk (orange) = *discount rate risk*

Decomposition of conditional CAPM alphas by innovation sensitivity: parametrization I



e risk (green) = *cash flow risk*
w risk (orange) = *discount rate risk*

- Campbell and Vuolteenaho (2004) use the decomposition: market beta = cash flow news beta + discount rate news beta

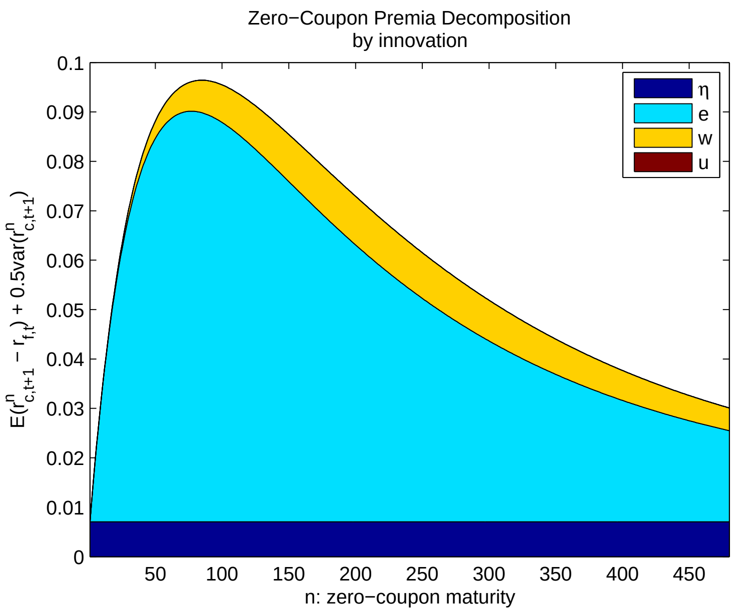
- In CV (2004), LW (2006): assets' risk premia reflect *high-price* cash flow risk, while beta/covariation is determined mostly by *low-price* discount rate sensitivity

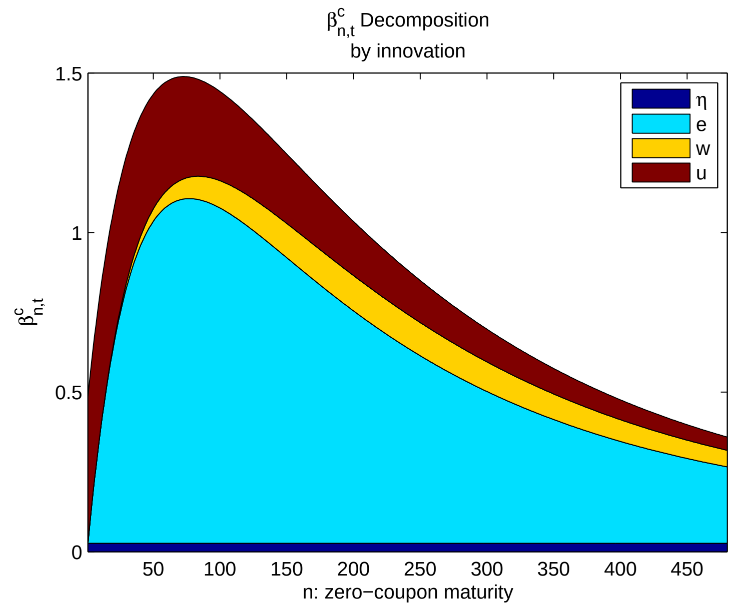
  - CAPM fails since beta mostly reveal discount rate risk

  - CV(2004) does well on cross-section, but has difficulty with the high level of the equity premium

- Can the CV mechanism be emulated here? Can we calibrate the model, match the equity premium, *and* get cash-flow risk to drive premia but discount rate risk to determine betas?

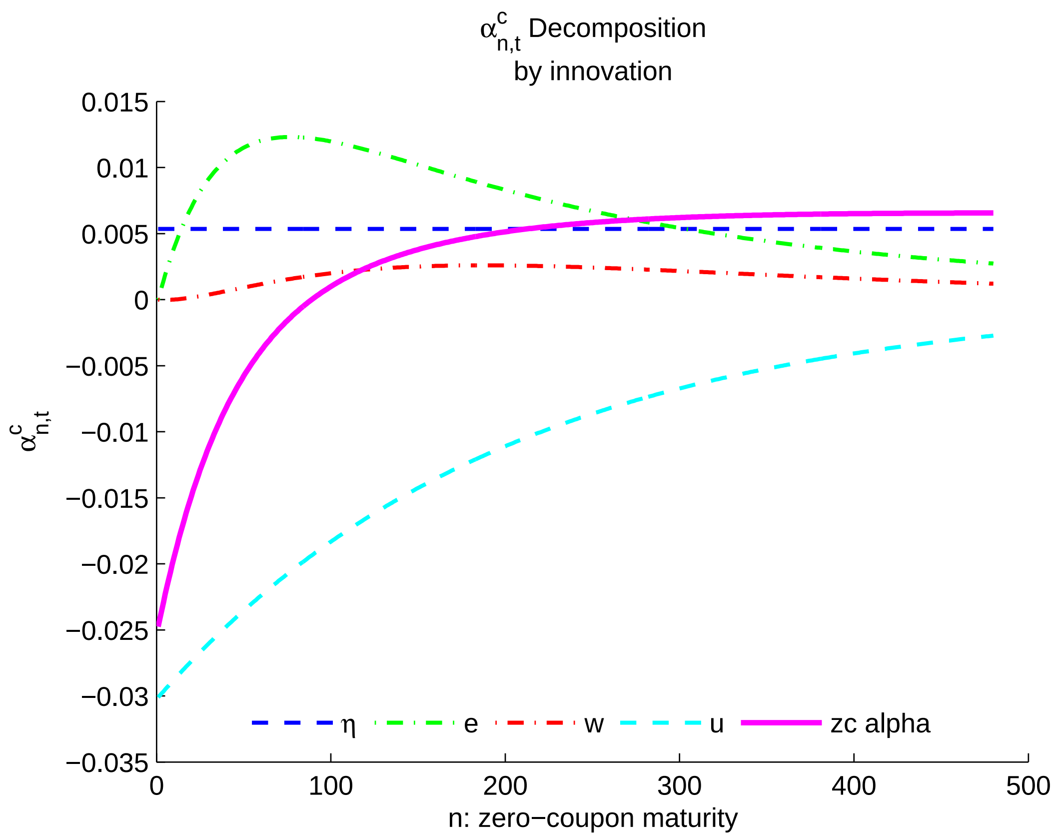
Decomposition of Term Structures for parametrization IV





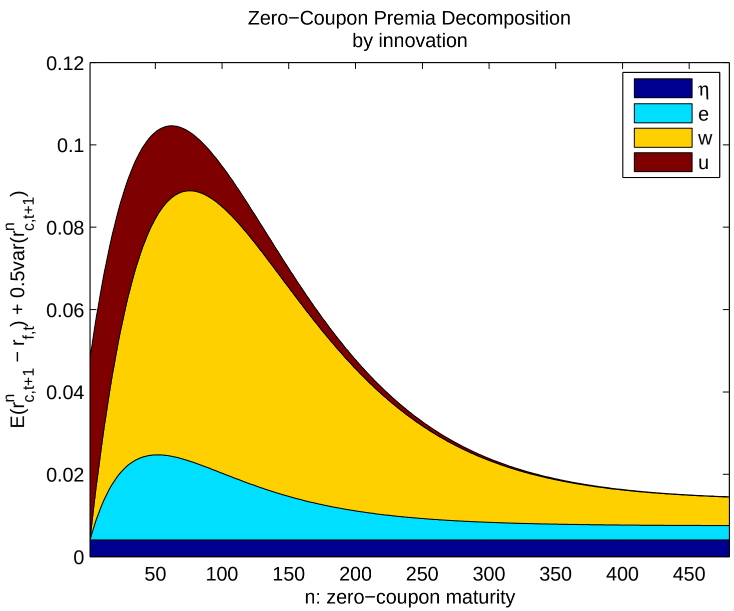
parametrization IV: $\rho_s = 0.995$, $\rho = 0.972$, $\sigma = 0.007$, $\phi_{sx} = 7$, $\varphi_e = 0.044$, $\varphi_d = 6$, $\nu = 0.987$, $\sigma_w = 3e - 006$, $\gamma = 12$, $\psi = 1.6$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

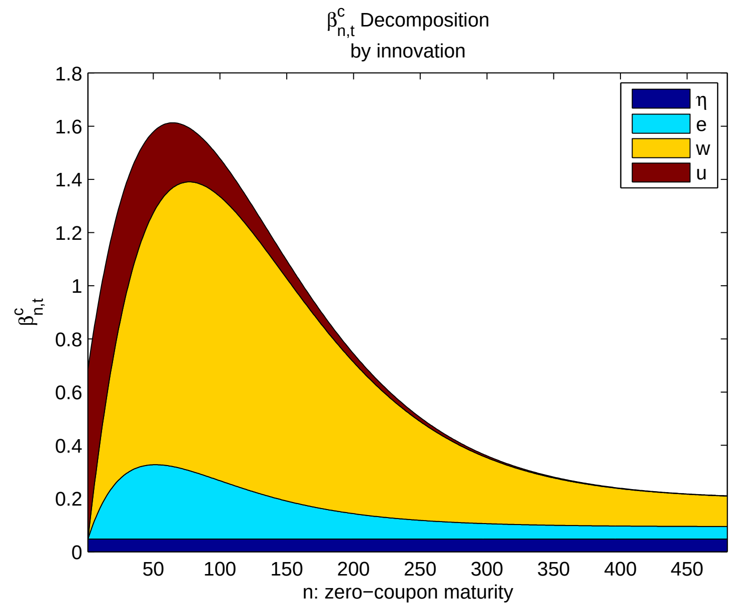
Decomposition of alphas for parametrization IV



parametrization IV: $\rho_s = 0.995$, $\rho = 0.972$, $\sigma = 0.007$, $\phi_{sx} = 7$, $\varphi_e = 0.044$, $\varphi_d = 6$, $\nu = 0.987$, $\sigma_w = 3e-006$, $\gamma = 12$, $\psi = 1.6$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0$

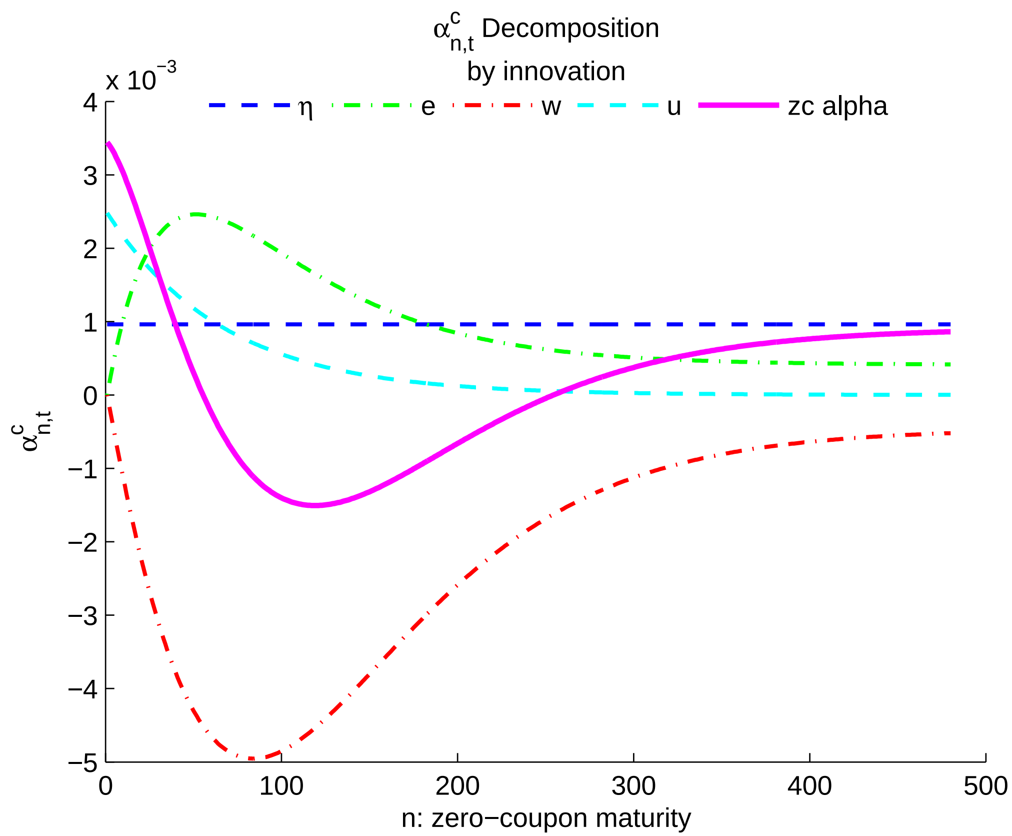
Decomposition of Term Structures for parametrization V





parametrization V: $\rho_s = 0.985$, $\rho = 0.972$, $\sigma = 0.0053$, $\phi_{sx} = 4$, $\varphi_e = 0.044$, $\varphi_d = 7$, $\nu = 0.98$, $\sigma_w = 1.2e - 005$, $\gamma = 12$, $\psi = 1.6$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.5$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0.73$

Decomposition of alphas for parametrization V



parametrization V: $\rho_s = 0.985$, $\rho = 0.972$, $\sigma = 0.0053$, $\phi_{sx} = 4$, $\varphi_e = 0.044$, $\varphi_d = 7$, $\nu = 0.98$, $\sigma_w = 1.2e - 005$, $\gamma = 12$, $\psi = 1.6$, $\mu_c = 0.0016$, $\delta = 0.999$, $\chi = 0.0006$, $\rho_{\eta u} = 0.5$, $\rho_{\eta e} = 0$, $\rho_{eu} = 0.73$

Conclusion

- changing the balance between cash-flow risks and discount-rate risks does *not* drive a wedge between the main driver of covariation/volatility and the primary determinant of risk premia

- the large zero-coupon strip alphas in LW (2006) and CLL (2006) do not appear to be feasible in this model
  - firms, *modeled as in LW (2006) and CLL (2006)*, will have small CAPM alphas

Further Work? Enriching the Model of Firms

- to be compatible with large CAPM alphas, the model of *firms* must be enriched
  - here a firms' fractional share of future dividend strips is *deterministic* (known at time t)

- Allow growth in a firm's fractional share to be *stochastic*
  - growth in a firm's fractional share of dividends could then be sensitive to economic conditions
  - i.e. heterogeneity in (log) dividend growth's exposure to consumption growth (as in BDL(2005), Kiku(2006))