What's Vol Got to Do With It

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Overview

- Infrequent but influential events can have a substantial effect on financial markets and asset prices
- Time variation in the risk of these events can move markets and cause variation in risk premia

Jump Risks, the VIX, and Return Predictability:

- The risk of infrequent jumps in long-run cash-flow growth and in volatility is important for equilibrium asset pricing
- A VIX-related quantity, the variance premium, reveals time variation in these jump risks and has significant return predictability

Summary of Results

- Theoretically show the link between time variation in uncertainty and jump risks to the economy, and the VIX's ability to predict returns.

- A consumption-based model with Long-Run Risks and rich uncertainty dynamics can account for
  1. Standard Asset Pricing moments of returns and cash flows
  2. Not-so-standard moments of VIX/volatility
  3. Short-run predictability by VIX/variance premium

- Non-Gaussian transient dynamics are important for capturing the uncertainty-related component of asset prices.

What is the VIX?

- The CBOE's Volatility Index
- Computed using the whole set of S&P 500 options maturing at a given date
- Based on recent work on model-free implied volatility
  - Jiang and Tian (2005), Carr and Wu (2007) generalize for general jump-diffusion

  These papers show that:

  $VIX_t^2 = E_t^Q \left[\int_t^{t+1} (dr)^2 \right]$— risk-neutral expectation of integrated variance over the next 30 days
- i.e. VIX_t^2 is the cost of a static option position that replicates the payoff of a variance swap

Variance Premium

$$(VP_t)\ \ Variance\ \ Premium_t = VIX_t^2 - \hat{\sigma}_t^2$$

- Is the difference of the (expected) integrated variance under the P (physical) and Q (risk-neutral) measures over the next month.
- Our $\hat{\sigma}_t^2$ comes from forecasting (S&P 500 futures) realized variance: $\sum_t^{t+1}(\Delta r)^2$

	VP(Fut–forecast)
Mean	11.20
Std.-Dev.	7.64
Minimum	3.27
Skewness	2.43
Kurtosis	12.16
AR(1)	0.65

- Note: This (ex-ante) quantity was always positive
- Positive skewness, large kurtosis

Variance Premium Predicts Returns: Monthly Regressions

Dependent	Regressors		OLS			Robust Reg.		
	X1	X2	β_1	β_2	$R^2(\%)$	β_1	β_2	$R^2(\%)$
r_{t+1}	VP_t		0.76		1.46	1.12		3.20
	(t-stat)		(2.18)			(2.77)		
r_{t+1}	VP_{t-1}		1.26		4.07	1.21		3.75
	(t-stat)		(3.90)			(2.97)		
r_{t+3}	VP_t		0.86		5.92	0.87		6.09
	(t-stat)		(3.19)			(4.12)		
r_{t+1}	VP_t	$\log(\mathrm{P/E})_t$	1.39	-48.67	8.30	1.81	-50.52	10.77
	(t-stat)		(3.00)	(-3.04)		(4.33)	(-4.36)	
r_{t+1}	VP_{t-1}	$\log(\mathrm{P/E})_t$	2.09	-58.12	13.43	1.98	-57.30	12.61
	(t-stat)		(4.82)	(-3.50)		(4.68)	(-4.85)	

- VP_t Regressor is not very persistent — no spurious regression/bias/std-error.
- Motivation that these R^2s can be economically significant:

  w/ timing max unconditional SR^2 increases by $\frac{1+s_0^2}{1-R^2}R^2 \approx R^2$ (Cochrane 1999)

  $\Rightarrow$ annualized SR increases from 0.54 to: 0.90 ($R^2 = 4.07\%$) 1.19 ($R^2 = 8.30\%$)
- R^2 comparison: dp ratio 1.5% detrended short rate 1.9% (Campbell, Lo, MacKinlay)

General Framework w/ Jumps

State vector: $Y_t \in \mathbb{R}^n$ with affine dynamics

$$
\begin{aligned}
Y_{t+1} &= \mu + FY_t + G_t z_{t+1} + J_{t+1} \\
& z_{t+1} \sim \mathcal{N}(0, \mathcal{I}) \\
& J_{t+1,i} = \sum_{j=1}^{N^i_{t+1}} \xi^j_i
\end{aligned}
$$

- N^i_{t+1} are (conditionally) independent Poisson counting processes
- The jump sizes ξ^j_i are i.i.d
- $\lambda_t = l_1 \sigma_t^2$ is the vector of jump intensities ($l_1 \in \mathbb{R}^n$)

- Preferences: Epstein-Zin

– γ is the coefficient of relative risk aversion

– ψ is the elasticity of intertemporal substitution

Dynamics: Long-Run Risks and Jumps

$$Y_{t+1} = \mu + FY_t + G_t z_{t+1} + J_{t+1}$$

Our calibrated specification:

$$Y_{t+1} = \left(\Delta c_{t+1}, x_{t+1}, \sigma^2_{t+1}, \Delta d_{t+1}\right)$$

$$\begin{aligned} E_t\left(\Delta c_{t+1}\right) &= \mu_c + x_t \\ E_t\left(x_{t+1}\right) &= \rho_x x_t \\ E_t\left(\sigma^2_{t+1}\right) &= E(\sigma^2_t) + \tilde{\rho}_\sigma(\sigma^2_t - E(\sigma^2_t)) \\ E_t\left(\Delta d_{t+1}\right) &= \mu_d + \phi x_t \end{aligned}$$

$$G_t G_t' = h + H_\sigma \sigma^2_t$$

Jumps are in x_t and σ^2_t: $\quad J_{t+1} = (0, J_{x,t+1}, J_{\sigma,t+1}, 0)$

$$\xi_x \sim \mathcal{N}(0, \sigma^2_x) \qquad \xi_\sigma \sim \Gamma(\nu_\sigma, \frac{\mu_\sigma}{\nu_\sigma})$$

Variance Premium: Main Component

- Let $r_{m,t+1}$ be the (log) return on the market
- The main component of the variance premium is:

$$\mathrm{var}_t^Q(r_{m,t+1}) - \mathrm{var}_t^P(r_{m,t+1})$$

To solve for this:

- Solve for Epstein-Zin pricing kernel (for jumps, use moment generating functions):

$$m_{t+1} - E_t m_{t+1} = -\Lambda' \left(G_t z_{t+1} + J_{t+1} - E_t(J_{t+1}) \right)$$

The price of risk vector Λ depends on preferences:

- For $\gamma = 1/\psi$ (CRRA), $\Lambda' = (\gamma, 0, 0, 0)$
- For $\gamma \neq 1/\psi$, $\Lambda = (\gamma, \Lambda_x, \Lambda_\sigma, 0)$
- We note importance of $\gamma > 1$, $\psi > 1$ for signs of risk prices
  - e.g. $\Lambda_\sigma < 0$

Variance Under P and Q

- Solve for $r_{m,t+1}$ as a function of Y_t and the shocks:

$$r_{m,t+1} - E_t[r_{m,t+1}] = B_r' G_t z_{t+1} + B_r'(J_{t+1} - E_t[J_{t+1}])$$

- Under P the conditional variance is:

$$\begin{aligned}\mathrm{var}_t^P(r_{m,t+1}) &= B_r' G_t G_t' B_r + \sum_i B_r^2(i)\mathrm{var}_t(J_{t+1,i}) \\ &= B_r' G_t G_t' B_r + B_r^{2\prime}\,\mathrm{diag}\left(\psi^{(2)}(0)\right)\lambda_t\end{aligned}$$

  - Note: $\psi_i(u)$ is the moment generating function of ξ_i

- Under Q the conditional variance is:

$$\mathrm{var}_t^Q(r_{m,t+1}) = B_r' G_t G_t' B_r + B_r^{2\prime}\,\mathrm{diag}\left(\psi^{(2)}(-\Lambda)\right)\lambda_t$$

Difference of Variances

- The main component of the variance premium is the difference:

$$\mathrm{var}_t^Q(r_{m,t+1}) - \mathrm{var}_t^P(r_{m,t+1}) = B_r^{2\prime} \mathrm{diag}\left(\psi^{(2)}(-\Lambda) - \psi^{(2)}(0)\right) \lambda_t$$

- $= 0$ when shocks are purely Gaussian since Gaussian components cancel
- $= 0$ for CRRA preferences since $\Lambda_x = \Lambda_\sigma = 0$
- Otherwise, *reveals* the latent jump intensity λ_t!
- For $\gamma > 1$, $\psi > 1$ and our jump specifications this is always *positive*

Predictability

- predictive regression:

$$r_{m,t+1} - r_{f,t} = \alpha + \beta_{\text{pred}} \left(\text{var}_t^Q(r_{m,t+1}) - \text{var}_t(r_{m,t+1}) \right) + \epsilon_{t+1}$$

- λ_t is in risk premia via σ_t^2
- $\text{var}_t^Q(r_{m,t+1}) - \text{var}_t^P(r_{m,t+1})$ "reveals" λ_t
- $\Rightarrow \text{var}_t^Q(r_{m,t+1}) - \text{var}_t^P(r_{m,t+1})$ has predictive ability
- $\gamma > 1$, $\psi > 1 \quad \Rightarrow \beta_{\text{pred}} > 0$
- variation in λ_t accounts for the predictive power

Calibration Results

Table VI

Model Calibration Results

Statistic	Data		Model 5%	Model 50%	Model 95%
Cashflow Dynamics					
$E[\Delta c]$	1.88	(0.32)	0.90	1.86	2.88
$\sigma(\Delta c)$	2.21	(0.52)	1.94	2.34	2.95
$AC1(\Delta c)$	0.43	(0.12)	0.26	0.46	0.64
$E[\Delta d]$	1.54	(1.53)	-1.58	1.74	5.65
$\sigma(\Delta d)$	13.69	(1.91)	11.04	13.23	15.72
$AC1(\Delta d)$	0.14	(0.14)	0.13	0.31	0.50
$corr(\Delta c, \Delta d)$	0.59	(0.11)	0.11	0.38	0.56
Returns					
$E[r_m]$	6.23	(1.96)	3.29	6.49	10.22
$E[r_f]$	0.82	(0.35)	0.52	1.08	1.53
$\sigma(r_m)$	19.37	(1.94)	16.30	19.42	23.90
$\sigma(r_f)$	1.89	(0.17)	0.80	1.22	2.38
$E[p-d]$	3.15	(0.07)	2.98	3.05	3.13
$\sigma(p-d)$	0.31	(0.02)	0.13	0.17	0.22
$skew(r_m - r_f)$ (M)	-0.43	(0.54)	-0.99	-0.21	0.30
$kurt(r_m - r_f)$ (M)	9.93	(1.26)	4.08	7.12	14.70
$AC1(r_m - r_f)$ (M)	0.09	(0.06)	-0.09	-0.01	0.06

Calibration Results

Table VI
Model Calibration Results

Statistic	Data		Model 5%	Model 50%	Model 95%
Variance Premium					
$\sigma(\text{var}_t(r_m))$	17.18	(2.21)	6.62	23.46	73.23
$AC1(\text{var}_t(r_m))$	0.81	(0.04)	0.66	0.82	0.92
$AC2(\text{var}_t(r_m))$	0.64	(0.08)	0.45	0.67	0.85
$E[VP]$	11.27	(0.93)	4.02	7.57	17.63
$\sigma(VP)$	7.61	(1.08)	3.00	10.65	33.23
$skew(VP)$	2.39	(0.59)	1.84	3.36	5.36
$kurt(VP)$	12.03	(3.30)	6.52	15.74	38.00
$\beta(1)$	0.76	(0.35)	-0.39	0.83	2.63
$R^2(1)$	1.46	(1.52)	0.02	1.94	9.73
$\beta(3)$	0.86	(0.27)	-0.27	0.76	2.09
$R^2(3)$	5.92	(4.67)	0.04	4.21	23.80
$\beta(6)$	0.49	(0.24)	-0.38	0.55	1.68
$R^2(6)$	3.97	(4.74)	0.07	5.66	33.64

Calibration Comparative Statics: Shutting Off Jumps

Table IX

Model Calibration Results

Statistic	Data		Model 1-A 5%	Model 1-A 50%	Model 1-A 95%	Model 1-B 5%	Model 1-B 50%	Model 1-B 95%	Model 1-C 5%	Model 1-C 50%	Model 1-C 95%
Variance Premium											
$\sigma(\text{var}_t(r_m))$	17.18	(2.21)	4.64	6.22	9.44	3.40	10.51	32.21	3.45	4.76	7.15
$AC1(\text{var}_t(r_m))$	0.81	(0.04)	0.79	0.87	0.93	0.65	0.81	0.924	0.81	0.87	0.93
$AC2(\text{var}_t(r_m))$	0.64	(0.08)	0.63	0.75	0.86	0.43	0.66	0.84	0.64	0.75	0.87
$E[VP]$	11.27	(0.93)	0.22	0.30	0.40	0.23	0.41	1.10	0.02	0.02	0.03
$\sigma(VP)$	7.61	(1.08)	0.12	0.17	0.25	0.19	0.59	1.81	0.01	0.01	0.02
$skew(VP)$	2.39	(0.59)	0.32	0.88	1.71	1.79	3.26	5.12	0.36	0.82	1.60
$kurt(VP)$	12.03	(3.30)	2.35	3.33	6.72	6.79	15.22	35.98	2.34	3.26	6.39
$\beta(1)$	0.76	(0.35)	-33.43	7.43	60.41	-14.94	4.31	28.52	-478.07	39.43	604.86
$R^2(1)$	1.46	(1.52)	0.00	0.22	2.42	0.01	0.61	5.89	0.01	0.20	1.96
$\beta(3)$	0.86	(0.27)	-32.28	6.19	62.27	-13.83	3.27	21.25	-485.47	39.43	604.86
$R^2(3)$	5.92	(4.67)	0.00	0.50	7.13	0.01	1.39	10.70	0.01	0.53	5.24
$\beta(6)$	0.49	(0.24)	-33.47	4.44	47.04	-10.89	2.59	16.30	-447.01	29.35	522.27
$R^2(6)$	3.97	(4.74)	0.016	1.11	12.62	0.01	1.74	15.83	0.01	1.04	9.11

Calibration Overview

- Matches time-averaged annual moments of consumption and dividends
- standard unconditional moments: the equity premium and risk free rate
- nonstandard moments: conditional volatility, variance premium, and predictive regressions
- compare data to model-based finite sample statistics with the same sample length as the data counterpart

Parameter highlights:

- $\gamma = 10, \ \ \psi = 2.0$
- x: $\rho_x = 0.975, \ l_{1,x} = 0.75/12$, jump's $\sigma_x = 2.5\times$ gaussian std. dev
- σ^2: $\tilde{\rho}_\sigma = 0.8975, \ l_{1,\sigma} = 0.75/12, \ v_\sigma = 1.0, \ \mu_\sigma = 2.5$

Note: $l_1 \times 12$ is average number of jumps per year